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A Stroboscopic Method in the Cylindrical Phase Space

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1. Introduction

In this paper, we are concerned with a real system of $n+1$ nonlinear differential equations of the form as follows:

$$(1.1) \qquad \begin{cases} \dfrac{dx_i}{dt}=\varepsilon X_i(x,\ \theta,\ t,\ \varepsilon) & (i=1,\ 2,\cdots,\ n), \\ \dfrac{d\theta}{dt}=\Theta(x,\ \theta,\ \varepsilon)+\varepsilon\Psi(x,\ \theta,\ t,\ \varepsilon), & \end{cases}$$

where

1° *ε is a parameter such that* $|\varepsilon|\ll 1$,

2° $X_i(x,\ \theta,\ t,\ \varepsilon)$ $(i=1,\ 2,\cdots,\ n)$, $\Theta(x,\ \theta,\ \varepsilon)$ *and* $\Psi(x,\ \theta,\ t,\ \varepsilon)$ *are twice continuously differentiable with respect to* $(x,\ \theta,\ \varepsilon)$ *in the domain*

$$D:\ |x|=\sum_{i=1}^{n}|x_i|<M,\ -\infty<\theta,\ t<+\infty,\ |\varepsilon|<\delta,$$

3° $X_i(x,\ \theta,\ t,\ \varepsilon)$ $(i=1,\ 2,\cdots,\ n)$ *and* $\Psi(x,\ \theta,\ t,\ \varepsilon)$ *are continuous with respect to t in the domain D and are periodic in t with period* $T_0>0$,

4° $X_i(x,\ \theta,\ t,\ \varepsilon)$ $(i=1,\ 2,\cdots,\ n)$, $\Theta(x,\ \theta,\ \varepsilon)$ *and* $\Psi(x,\ \theta,\ t,\ \varepsilon)$ *are periodic in θ with period* 2π,

5° $\Theta(x,\ \theta,\ 0)\neq 0$ *for any* $(x,\ \theta)\in D$.

The system of the form (1.1) cannot have any periodic solution of the proper sense, because $\theta(t)$ is monotonous due to the assumption 5°. But it may have a solution such that

$$(1.2) \qquad \begin{cases} x_i(t+lT_0)=x_i(t) & (i=1,\ 2,\cdots,\ n), \\ \theta(t+lT_0)=\theta(t)+2m\pi, & \end{cases}$$

where l and m are integers. Such a solution represents a closed curve in the cylindrical phase space, namely the space consisting of the points $(x,\ \theta)$, θ being considered modulus 2π. So the solution satisfying the condition (1.2) can be called a periodic solution in the cylindrical phase space. In the sequel,