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Dimension Theory on Relatively Semi-orthocomplemented Complete Lattices

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Introduction

In my previous paper [11], the dimension theory of a relatively semi-orthocomplemented complete lattice (cf. [13]) with an equivalence relation has been developed by the axiomatic treatement, which enables us to unify the dimension theories of the von Neumann algebras and the continuous geometries. Our method is very similar to that of Loomis [8], but he treated only the case where the lattice is orthocomplemented. Let L be a relatively semi-orthocomplemented complete lattice where the semi-orthogonality "$\perp$" satisfies the following condition: $a_\delta \uparrow a$, $a_\delta \perp b \Rightarrow a \perp b$. It has been shown in [11] that if there is an equivalence relation in L satisfying certain axioms (denoted by $(2, \beta)$—$(2, \zeta)$ in [11]) then there exist the dimension functions with respect to this equivalence relation. In [12], this system of axioms was modified for the purpose of giving simple conditions for a Baer *-ring under which the lattice of projections of this ring has the dimension functions with respect to the algebraic equivalence (or the *-equivalence) introduced by Kaplansky. Indeed, these conditions are satisfied by the Baer *-rings considered by Kaplansky [6] and [7], and consequently by the AW*-algebras and the von Neumann algebras.

Now, we consider the projectivity of an upper-continuous complemented modular lattice for the purpose of generalizing the dimension theory of the continuous geometries. The systems of axioms given in [11] and [12] include the axiom of (complete or finite) additivity, but the above projectivity does not generally satisfy this axiom. For this reason, in this paper we shall give another system of axioms which is weaker than the systems in [12] and [8], and we shall develop the dimension theory on L, which not only covers the existing dimension theories of the Baer *-rings and the continuous geometries but also throws light on the dimension theory of upper-continuous complemented modular lattices.

In this paper, the system of axioms for equivalence relation is given as follows:

(A_1) $a\sim 0$ implies $a=0$;

(A_2) if $a\sim b_1 \dot{\cup} b_2$ then there exists a decomposition $a=a_1 \dot{\cup} a_2$ with $a_i\sim b_i$ $(i=1, 2)$;

(B) if we put $a=(a\cap b)\dot{\cup} a_1$, $b=(a\cap b)\dot{\cup} b_1$, $a\cup b=a_2\dot{\cup} b=a\dot{\cup} b_2$ for any a, b,