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Maximum of the Amplitude of the Periodic Solution of van der Pol's Equation

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1. Introduction

Previously, by Urabe and his collaborators [3, 5, 6][1], the periodic solutions of van der Pol's equation

$$\frac{d^2x}{dt^2}-\lambda(1-x^2)\frac{dx}{dt}+x=0 \quad (\lambda>0) \tag{1.1}$$

have been computed for various values of λ up to 20. One of the important facts found by their computation is the behavior of the amplitude of the periodic solution as the damping coefficient varies from 0 to infinity. The amplitudes a obtained for various values of λ are as follows:

Table 1

λ	a	λ	a
0	2.000	6	2.0199
1	2.009	8	2.0169
2	2.0199	10	2.0145
3	2.0235	20	2.0077
4	2.0231		
5	2.0216	∞	2.0000

As was pointed out by Urabe [4], the above value for $\lambda=10$ differs by only 0.0007 from that given by the asymptotic expression of Дородницын [1]

$$a=2+\frac{\alpha}{3}\lambda^{-4/3}-\frac{16}{27}\frac{\log\lambda}{\lambda^2}+\frac{1}{9}(3b_0-1+2\log 2 \tag{1.2}$$

$$-8\log 3)\frac{1}{\lambda^2}+O(\lambda^{-8/3})$$

$$(\alpha=2.338107,\ b_0=0.1723)$$

and the value for $\lambda=20$ coincides exactly with that given by the above asymptotic expression. From this, it may be supposed that, for λ greater than 10, the amplitude behaves quite approximately in accordance with the asymptotic

1) The numbers in square brackets refer to the references listed at the end of the report.