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On a Space of Distributions with Support in a Closed Subest

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For any closed subset F of R^n, the n-dimensional Euclidean space, $\mathscr{D}'_F$ denotes the space of all the distributions with support in F. As well known, the space $\mathscr{D}'$ of the distributions on R^n is complete and bornological, hence barrelled, and the space $\mathscr{D}'_F$ is the closed subspace of $\mathscr{D}'$. In general, a closed subspace of a bornological (resp. barrelled) space is not always bornological (resp. barrelled). In this paper it is shown that $\mathscr{D}'_F$ is bornological and barrelled for any closed subset F. Spaces of this type are often encountered in the applications of the theory of distributions. We shall also be concerned with constructive structure of $\mathscr{D}'_{\Gamma_0}$, Γ_0 being the first quadrant of R^n, because of its importance in symbolic calculus [9].

In this paper we shall use the notations of L. Schwartz [6] without any further reference.

1. Preliminaries. For our later purpose we need the following lemma of Hirata [4]: Let E be the projective limit of a sequence of bornological spaces E_j with epijective continuous mappings $\pi_{j,j+1}: E_{j+1}\rightarrow E_j,$ then E is bornological under the condition that for any bouned subset B_j of E_j there exists a bounded subset B of E such that $\pi_j(B)=\mathrm{B}_j$, where π_j is the mapping of E onto E_j determined by $\{\pi_{j,j+1}\}$. It is to be noticed that this condition is satisfied if each E_j is a Silva space in the sense of Yoshinaga [10], that is, the dual space of a Schwartz (F) space. To see this, we first show

Proposition 1. *If ϕ is an epijective continuous mapping of a Silva space H onto another G, then ϕ is epimorphic and any bounded subset of G is an image of a bounded subset of H.*

Prooe. Any Silva space is reflexive and is the dual of a Schwartz (F) space. Therefore we may consider H (resp. G) as the dual of a Schwartz (F) space K (resp. L). As ϕ is onto, it becomes the dual mapping of a monomor phism ψ of L into K [1]. Then any equicontinuous subset of L' $(=G)$ is a ϕ-image of an equicontinuous subset of $K'(=H)$ [2]. Since the space of type (F) is a barrelled space, any equicontinuous subset of K' (resp. L') is a bounded subset of H (resp. G) and vice versa. As ϕ is onto, it is epimorphic [10]. The proof is complete.

Let B_j be any bounded subset of E_j. We put $B_i=\pi_{i,i+1}\circ\cdots\circ\pi_{j-1,j}(B_j)$ for