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Stochastic Games with Infinitely Many Strategies

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Introduction

A stochastic game is originated by L. S. Shapley [1]. It is a game consisting of a finite collection of positions among which two players 1 and 2 proceed by steps from position to position according to certain prescribed transition probabilities jointly controlled by them: there is assumed a finite number of positions 1, 2,$\cdots$, N, and at the position k, a game Γ_k is played, in which Player 1 can choose any strategy among the given m_k pure strategies and Player 2 can also choose any strategy among the given n_k pure strategies. We assume that, at the position k, the players 1 and 2 choose their i-th and j-th alternatives respectively. We also assume that the game stops with the probability $p_{k0}^{ij}>0$ and the game moves to another position l with the probability p_{kl}^{ij}. Thus the game may not be bounded in length. Player 1 takes the gain g_k^{ij} from Player 2 whenever the pair i, j of pure strategies is chosen at the position k. In Shapley's stochastic game, both players use the so-called stationary strategies, namely at the position k, whenever and by whatever route the position may be reached, the probability distributions of choosing pure strategies are specified. And payments accumulate throughout the course of the play. Let $\vec{\Gamma}_k$ denote the infinite game begun with Γ_k. With the aid of dummy games, Shapley gave a method of finding the value of the stochastic game which is the collection $\boldsymbol{\Gamma}=\{\vec{\Gamma}_k, k=1, 2,\cdots, N\}$.

As stated above, L. S. Shapley has assumed that at each position there are only finite numbers of pure strategies from among which each player can choose one. Generalizations of his theory to infinite sets of alternatives seem yet to be obtained although he has promised in [1] to discuss them in another place. It seems interesting to generalize his theory to infinite sets of pure strategies or to an infinite number of positions.

After giving preliminary remarks in Section 1, we proceed, in Section 2, to the definition of the stochastic game, at each position of which each player may choose any one out of infinite pure strategies. With suitably imposed conditions on pay-offs and transition probabilities, we show that the stochastic games thus defined are strictly determined (Theorem 1 below). Some considerations are given centering around the ϵ-optimal strategies. The proof of the theorem 1 is carried out with the aid of the dummy games associated with the original stochastic game (Lemmas 3, 4).

In final Section 3, we concern ourselves with the stochastic games with