J. Sci. Hiroshima Univ. Ser. A-I
26 (1962), 111-122

The Potential Force Yielding a Periodic Motion with Arbitrary Continuous Half-Periods

Minoru Urabe

(Received September 15, 1962)

1. Introduction

In the previous papers [3, 4], the author has given a method to determine the potential force $g(x)$ so that the period of the periodic solution of the equation

$$\frac{d^2x}{dt^2}+g(x)=0 \tag{1.1}$$

may be an arbitrary given continuous function of the amplitude of the velocity or an arbitrary given continuously differentiable function of the amplitude.

Let R be the maximum velocity (i.e. the amplitude of the velocity) and let a and b be respectively the positive maximum and negative minimum displacement of x. Let $\tau_1/2$ and $\tau_2/2$ be respectively the times required to reach the state of the positive maximum displacement a and the state of the negative minimum displacement b from the equilibrium point $x=0$.

In his paper [1], Z. Opial called the quantities τ_1 and τ_2 respectively the positive half-period and the negative half-period, and discussed the various relations between these half-periods and the potential force $g(x)$.

The half periods τ_i $(i=1, 2)$ are the functions of R. Further the positive half-period τ_1 and the negative half-period τ_2 are also respectively the functions of the positive maximum displacement a and the negative minimum displacement b.

Opial has proved under very mild conditions that $g(x)$ is uniquely fixed if $\tau_1=\hat{\tau}_1(a)$ and $\tau_2=\hat{\tau}_2(b)$ are given. But he has not given a method to determine $g(x)$ for which any solution of (1.1) has the given arbitrary $\hat{\tau}_1(a)$ and $\hat{\tau}_2(b)$.

In the present paper, by means of the techniques used in his previous papers [2, 3, 4], the author will give a method to determine $g(x)$ so that either

1° τ_1 and τ_2 may be respectively arbitrary given continuous functions $\tilde{\tau}_1(R)$ and $\tilde{\tau}_2(R)$, or

2° τ_1 and τ_2 may be respectively arbitrary given continuously differentiable functions $\hat{\tau}_1(a)$ and $\hat{\tau}_2(b)$ whose derivatives fulfill the Lipschitz condition.

The problem of "*tautochronism*" [1] is to determine $g(x)$ so that $\tilde{\tau}_1(R)$ and