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# *The Potential Force Yielding a Periodic Motion whose Period is an Arbitrary Continuously Differentiable Function of the Amplitude*

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## 1. Introduction

In the previous paper [2], the author has given a method to determine the potential force $g(x)$ so that the period of the periodic solution of the equation

$$\frac{d^2x}{dt^2}+g(x)=0 \tag{1.1}$$

may be an arbitrary given continuous function of the amplitude of the velocity.

In the present paper, first, we shall give a method to determine $g(x)$ so that the period of the periodic solution of (1.1) may be an arbitrary given continuously differentiable function $\omega_1(a)$ of the positive maximum displacement $a$ of $x$ whose derivative $\omega_1'(a)$ with respect to $a$ satisfies the Lipschitz condition. Our method is based on solution of a certain integral equation to which the problem is reduced by the techniques used in the previous paper [2].

Next there will be given a method to determine the desired potential force $g(x)$, namely $g(x)$ such that the period of the periodic solution of (1.1) may be an arbitrary given continuously differentiable function $\omega(A)$ of the amplitude $A$ whose derivative $\omega'(A)$ with respect to $A$ satisfies the Lipschitz condition. By the same techniques as in the first problem, the present problem is reduced to solution of an integral equation which is of a particular type of the integral equation solved already in the first problem.

Lastly, in illustration of our method, there will be given a potential force $g(x)$ such that the period $\omega$ of the periodic solution of (1.1) is a linear function of the amplitude $A$.

Since the work of the present paper is based on the main theorem in the previous paper [2], it is restated here for the convenience of the readers.

Theorem 0. *In case $g(x)$ is continuous in the neighborhood of $x=0$ and differentiable at $x=0$, if any solution of the equation* (1.1) *near $x=\dot{x}=0$ ($\cdot = d/dt$) oscillates around $x=\dot{x}=0$ with a bounded period, then*