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Error Estimation in Numerical Solution of Equations by Iteration Process

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1. Introduction

Let R be a linear normed space and F be a complete subset of R. Let f be a functional defined on F such that $f(F)\subset R$.

We assume that

(i)

$$\|f(x')-f(x'')\|\leqq K_0\|x'-x''\| \tag{1.1}$$

for any $x', x''\in F$, *where*

$$0<K_0<1; \tag{1.2}$$

(ii)

$$\|f^*(x)-f(x)\|\leqq\varepsilon \tag{1.3}$$

for any $x\in F$, *where* $f^*(x)$ *is a numerical valuation of* $f(x)$ *in actual computation with the error bound* ε (>0) *such that* $f^*(F)\subset R$ *(here, by a numerical valuation in actual computation, we mean a valuation by a set of finite numbers of the numbers rounded to a certain fixed number of decimal digits)*;

(iii) *for a certain numerical value* $x_0\in F$,

$$\sum\left\{h:\|h-x_1^*\|\leqq\frac{K_0}{1-K_0}\|x_1^*-x_0\|+2\delta_0\right\}\subset F, \tag{1.4}$$

where $x_1^*=f^*(x_0)$ *and*

$$\delta_0=\frac{\varepsilon}{1-K_0}. \tag{1.5}$$

Then, by the author's previous paper [1],

(i) *the equation*

$$x=f(x) \tag{1.6}$$

has one and only one solution in F;

(ii) *the unique solution* $\bar{x}$ *of* (1.6) *is obtained by the ideal iteration process*