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Bifurcation of a Periodic Solution of van der Pol's Equation with the Harmonic Forcing Term

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1. Introduction.

In this paper, we are concerned with van der Pol's equation with harmonic forcing term

$$\frac{d^2x}{dt^2} - \varepsilon(1-x^2)\frac{dx}{dt} + x = \varepsilon E \sin \omega t \quad (E \neq 0), \tag{1.1}$$

where ω is a number close to unity. Here, as is readily seen, we may assume $\varepsilon > 0$ and $E > 0$ without loss of generality. By the substitution of the time variable, the equation (1.1) can be written in the form as follows:

$$\frac{d^2x}{dt^2} + x = \varepsilon\left\{-Ax + (1-x^2)\frac{dx}{dt} + E\sin t\right\}, \tag{1.2}$$

which is rewritten in a simultaneous form as follows:

$$\begin{cases} \dfrac{dx}{dt} = y, \\[2mm] \dfrac{dy}{dt} = -x + \varepsilon\,\{E\sin t - Ax + (1-x^2)y\}. \end{cases} \tag{1.3}$$

In (1.2) or (1.3), notice that, besides ε, the quantities A and E are also supposed to be the parameters.

By the linear transformation

$$\begin{cases} x = \xi\cos t + \eta\sin t, \\ y = -\xi\sin t + \eta\cos t, \end{cases} \tag{1.4}$$

the system (1.3) is transformed to the system of the form as follows:

$$\begin{cases} \dfrac{d\xi}{dt} = \varepsilon f(\xi, \eta, t), \\[2mm] \dfrac{d\eta}{dt} = \varepsilon g(\xi, \eta, t), \end{cases} \tag{1.5}$$

where