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Reduction of Group Varieties and Transformation Spaces

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In the paper [3], Koizumi and Shimura solved affirmatively the following problem: *let A and B be abelian varieties defined over a field k with a prime divisor $\mathfrak{p}$. Suppose that there exists a homomorphism of A onto B, defined over k. If A is without defect for $\mathfrak{p}$, then is there an abelian variety which is isomorphic to B over k and without defect for $\mathfrak{p}$?* In this paper we shall generalize this result for the cases of arbitrary group varieties and homogeneous spaces (Theorem 3), and apply it to a problem which concerns compatibility of the reduction process with the process making a coset space of a group variety by a subgroup (Theorem 4). Our generalization is not complete, because we need a ground ring extension in the process of constructing a group $\mathfrak{p}'$-variety (resp. a homogeneous $\mathfrak{p}'$-space) from a pre-group $\mathfrak{p}$-variety (resp. a pre-homogeneous $\mathfrak{p}$-space). However if k is complete with respect to the prime $\mathfrak{p}$, we do not need any ground ring extension. In other words it is possible to generalize completely the result obtained in [3] in this case.

First we shall define a pre-group $\mathfrak{p}$-variety, a pre-transformation $\mathfrak{p}$-space, etc., which corresponds to a pre-group, a pre-transformation space, etc. in [9], and prove some basic results (§1). Next Weil's idea in [11] is adapted to the case of $\mathfrak{p}$-simple $\mathfrak{p}$-varieties. The main result of §2 is stated in Theorem 1, whose applications will be seen in §3. Then we shall apply Weil's method of construction of a group variety (resp. a transformation space) from a pre-group (resp. a pre-transformation space) to the case of $\mathfrak{p}$-simple $\mathfrak{p}$-varieties. Theorem 2 in §3 corresponds to the main theorem in [9]. Theorem 3 is, then, a direct consequence of the basic results in §1 and Theorem 2. In §4 an application of Theorem 3 is given, to which we referred already in the above. §5 is devoted to the study of the reduction of generalized Jacobian varieties under a certain restriction.

Throughout the paper, we shall fix the basic field k and a discrete valuation ring $\mathfrak{o}$ with the maximal ideal $\mathfrak{p}$ and denote by κ the residue class field $\mathfrak{o}/\mathfrak{p}$. The terminologies and the notations in [8] and [13] will be freely used.

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§1. Group $\mathfrak{p}$-varieties and homogeneous $\mathfrak{p}$-spaces.

Let $(V, \bar{V})$ and $(W, \bar{W})$ be two $\mathfrak{p}$-simple $\mathfrak{p}$-varieties[1]), and let f be a rational

1) We shall denote $\mathfrak{p}$-varieties by $(V, \bar{V})$ etc.. For the precise notations, see §5 in [13]. A $\mathfrak{p}$-variety is called to be $\mathfrak{p}$-simple, if the corresponding model of a function field is $\mathfrak{p}$-simple.