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On θ-convolutions of vector valued distributions

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Introduction

In our previous paper [4] collaborated with Y. Hirata, we have introduced the notion of ι-convolution of vector valued distributions, which is a natural extension of the notion of the usual convolution of scalar valued distributions. On the other hand, in the Schwartz theory of convolution [11] [12], the problem concerning convolution has been worked out from a different standpoint. For instance, let $\mathscr{H}$, $\mathscr{K}$ and $\mathscr{L}$ be three normal spaces of distributions on R^N, N-dimensional Euclidean space. Let $\cup: \mathscr{H}\times\mathscr{K}\to\mathscr{L}$ be a bilinear map which is hypocontinuous with respect to the bounded subsets of $\mathscr{H}$ and $\mathscr{K}$. Let E, F, G be three Banach spaces and $\theta: E\times F\to G$ be a continuous bilinear map. He asked the question whether it would be possible to define a unique bilinear map $\cup_\theta: \mathscr{H}(E)\times\mathscr{K}(F)\to\mathscr{L}(G)$ in such a way that the map satisfies the following conditions:

(a) $\cup_\theta$ is hypocontinuous with respect to the bounded subsets of $\mathscr{H}(E)$ and $\mathscr{K}(F)$.

(b) For decomposed elements, that is, for elements of the type $S\otimes e$ and $T\otimes f$ of $\mathscr{H}(E)$ and $\mathscr{K}(F)$ we have

$$(S\otimes e)\cup_\theta(T\otimes f)=(S\cup T)\otimes\theta(e,f).$$

Under certain plausible conditions imposed on $\mathscr{H}$, $\mathscr{K}$ and $\mathscr{L}$, the problem has been settled definitely. Consider the convolution map $*: \mathscr{H}\times\mathscr{K}\to\mathscr{L}$ which is, by definition [12], a separately continuous map coinciding on $\mathscr{D}\times\mathscr{D}$ with the usual convolution. Suppose that $*$ is hypocontinuous with respect to the bounded subsets of $\mathscr{H}$ and $\mathscr{K}$. Now if we take the map $*$ for the map $\cup$ above, then the problem just described turns out to be the one concerning the convolution. Although his theory sheds a new light on the basic operations of vector valued distributions, there remains something to be desired as to the convolution maps:

(1) Since the map $*$ need not agree with the usual convolution (the example is given in [14]), $S*T$ may have only relative meanings and *a fortiori* the same for $\vec{S}*_\theta\vec{T}$.

(2) Even if the $*$ agrees with the usual convolution, $\vec{S}*_\theta\vec{T}$, considered as convolution of two vector valued distributions, has no intrinsic meanings, but may depend on $\mathscr{H}(E)$ and $\mathscr{K}(F)$ in which $\vec{S}$ and $\vec{T}$ are contained respectively.