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Remarks to Accelerated Iterative Processes for Numerical Solution of Equations

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1. Introduction.

In the present paper, we are concerned with error estimation for some accelerated iterative processes for numerical solution of equations, the round-off errors committed in actual computation being taken into consideration. From our results, some remarks will be made on the use of these formulas in actual computation.

For simplicity, we suppose the equation is given in the form

$$x=\varphi(x), \tag{1.1}$$

where $\varphi(x)$ is continuously differentiable in the closed interval I.

We assume that

$$0 \leqq |\varphi'(x)| \leqq K<1 \quad \text{in } I \tag{1.2}$$

and that

$$S\{h:\ |h-x_1| \leqq \frac{K}{1-K}\ |x_1-x_0|\} \subset I \tag{1.3}$$

for $x_0 \in I$ and $x_1=\varphi(x_0)$. Then it is well known [2] that

1° the iterative process

$$x_{n+1}=\varphi(x_n) \quad (n=0, 1, 2, \cdots) \tag{1.4}$$

can be continued indefinitely so that $x_n \in S$ $(n=1, 2, \cdots)$;

2° the sequence $\{x_n\}$ $(n=0, 1, 2, \cdots)$ converges in S and $\lim\limits_{n\to\infty} x_n=\bar{x}$ satisfies the equation (1.1);

3° $x=\bar{x}$ is the unique solution of (1.1) in I.

As is well known, the convergence of this classical iterative process (1.4) (in what follows this process is abbreviated as CI-process) is not fast when K is not small. To rescue this fault, some methods are devised to accelerate the speed of the convergence of the above iterative process.

One of these accelerated processes is Aitken's δ^2 process [1, 3, 4, 5]. It is based on using the function