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Weak Domination Principle

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Introduction and Preliminaries

In a paper [1] on "Modèles finis" of the potential theory, Choquet and Deny obtained the following interesting result. If a positive kernel on a space of a finite number of points is non-degenerate and satisfies the weak balayage (or equivalently domination) principle, then it satisfies the ordinary balayage (domination) principle or the inverse balayage (domination) principle. In this paper we shall extend this result to a positive continuous (in the extended sense) kernel on a locally compact Hausdorff space. Similar extension was tried by Ninomiya [4] for positive continuous symmetric kernels. His result states that if a positive symmetric kernel is of positive [negative resp.] type and satisfies the weak balayage principle, then it satisfies the ordinary [inverse resp.] balayage principle.

Let Ω be a locally compact Hausdorff space and G be a positive continuous (in the extended sense) kernel on Ω such that $G(x, y)$ is finite at any point $x \neq y$. Throughout this paper we assume that every compact subset of Ω is separable[1], and we shall use the same notations as in the author's paper [2].

First we define domination principles which we shall consider in this paper.

(I) *Weak domination principle.* If $G\mu \leq G\nu$ on $S\mu \cup S\nu$ for $\mu \in \mathfrak{E}_0$ and $\nu \in \mathfrak{M}_0$, then the same inequality holds in Ω[2].

(II) *Ordinary domination principle* (or simply, *domination principle*). If the above inequality holds on $S\mu$, so it does in Ω.

(III) *Inverse domination principle.* If the above inequality holds on $S\nu$, so it does in Ω.

(IV) *Elementary domination principle.* If $aG(x_1, x_1) \leq G(x_1, x_2)$ with $a > 0$, then $aG(z, x_1) \leq G(z, x_2)$ for any point z in Ω.

(V) *Elementary inverse domination principle.* If $aG(x_2, x_1) \leq G(x_2, x_2)$ with $a > 0$, then $aG(z, x_1) \leq G(z, x_2)$ for any point z in Ω.

(VI) *Strong elementary domination principle.* If $G\mu \leq G\varepsilon_{x_0}$ on $S\mu$ for $\mu \in \mathfrak{E}_0$ and $x_0 \notin S\mu$, then $G\mu \leq G\varepsilon_{x_0}$ in Ω.

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1) All the results in this paper hold with a slight modification for positive continuous kernels on Ω compact subsets of which are not necessarily separable (cf. Nakai [3]).

2) $\mathfrak{M}_0$ is the totality of positive measures with compact support and $\mathfrak{E}_0$ is the totality of positive measures in $\mathfrak{M}_0$ with finite energy.