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On the Multiplicative Products of Distributions

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In the theory of distributions of L. Schwartz [6], multiplication for two distributions leads to difficulties. Schwartz [6] has observed that the multiplicative product is well defined if locally one is "more regular" than the other is "irregular". An approach to define multiplication for distributions has been made by Y. Hirata and H. Ogata [2]. In like manner J. Mikusiński [5] has also given a definition of multiplication. The main purpose of this paper is to show that these two definitions lead to equivalence (§1. Theorem). §2 is devoted to the discussions on the multiplicators of normal spaces of distributions. We show that, in case of functions, the ordinary product is not in general the product in the above sense even if it is a function. In §1 and §3 we make some remarks on the exchange formula for Fourier transformation.

Throughout this paper we assume that unless otherwise specified a Euclidean space on which distributions are defined is the same N-dimensional space.

1. Multiplicative products. By a *δ-sequence* or a *sequence of regularizations* we understand every sequence of non-negative functions $\rho_n \in \mathcal{D}$ with the following properties:

(1) $\operatorname{Supp} \rho_n$ converges to 0 when $n \to \infty$;

(2) $\int \rho_n(x)dx=1$, the integral being extended to the whole N-dimensional space.

Given any distribution S and any δ-sequence $\{\rho_n\}$, the sequence $S_n=S*\rho_n$ will be called a *regular sequence* of S. Every regular sequence of S converges to S in $\mathcal{D}'$.

Recall the definitions of multiplication for two distributions S and T given by Y. Hirata and H. Ogata ([2], p. 150) and J. Mikusiński ([5], p. 254):

Definition 1 (Hirata and Ogata). *By $[S]T$ we understand the distributional limit of the sequence $\{S_nT\}$, if it exists for every regular sequence of S. Similarly for $S[T]$. If both $[S]T$ and $S[T]$ exist and coincide, then $[ST]=[S]T=S[T]$ is called a multiplicative product of S and T.*