J. Sci. Hiroshima Univ. Ser. A-I
28 (1964), 199-207

On Local Loops in Affine Manifolds

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(Received September 21, 1964)

1. Introduction.

Local loops[1] have been treated, in the di-associative case, by Malcev [4], and general properties of topological loops have been studied by Hofmann [2] and others.

In the present paper, we shall show that a differentiable manifold with an affine connection forms a local loop in a neighbourhood of each of its points, if a product operation of two points on it is defined by means of parallel displacement of geodesics.

Next, we shall lead the fact that in the manifold with symmetric affine connection, if the local loop constituted at a point is left di-associative, the curvature tensor vanishes at the point.

Moreover, we shall refer to a sufficient condition for the group of linear transformations of tangent space induced by right inner transformations of the local loop to coincide with the local holonomy group, at the unit element of the local loop.

In particular, both of them really coincide with each other in reductive homogeneous space with canonical affine connection.

The author wishes to express his hearty gratitude to Professor K. Morinaga for his many valuable suggestions and kind encouragement and to Mr. K. Miyashita for his co-operation during the preparation of this paper.

2. Local Loops in Affinely Connected Spaces.

Definition 1. A *local loop* $\mathfrak{L}(U, f)$ is a pair formed by a topological space U and a continuous mapping f of an open subset S of $U\times U$ into U, satisfying the following conditions:

(a) On each of subsets $S_x^1=S\cap(\{x\}\times U)$ and $S_y^2=S\cap(U\times\{y\})$, the mapping f is a local homeomorphism.

The image $f(x, y)$ of $(x, y)\in S$ is called the product of x and y, and denoted by xy.

(b) There exists an element e of U such that $(e, e)\in S$ and that $ey=y$ on

(1) See Definition 1.