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Note on First Recurrence and First Passage Times for Renewal Processes

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1. Introduction

We shall deal with the probability generating functions of the 'first recurrence times' and the 'first passage times' for renewal processes with discrete (time) parameters, which were treated by W. Feller [3], [4] and M. S. Bartlett [1], [2]. Bartlett obtained not only some results for the 'first recurrence time' probability generating functions due to Feller from a different point of view, but also the probability generating functions of the 'first recurrence times' and the 'first passage times' when some conditions are imposed on the paths of the processes. Though Bartlett's approch seems to be ingenious at a glance, it is too complicated to apply the method to more involved cases where several conditions are imposed on the paths of the processes.

We shall generalize Feller's approch, and systematically derive the 'first recurrence time' as well as the 'first passage time' probability generating functions when several conditions are imposed on the paths of the processes. The results obtained will cover the most complicated results due to Bartlett as their special cases, and one of them agrees with a formula due to J. H. B. Kemperman [5].

2. Notations

We shall deal with a process with possible states $a, b, c, \cdots$, or $1, 2, \cdots$, which has the following two properties:

(i) The process is temporally homogeneous;

(ii) Any one of the states (or the sets of states) considered has the renewal property.

For convenience, we denote various kinds of events, their probabilities and their probability generating functions treated in this paper as follows:

$\underset{S}{\pi^{ab}}(m+n|m)$: The event of the process being in state b at time $m+n$ $(n \geqq 1)$, given that the process is in state a at time m and *no intermediate passage* to any one of the states in the set S occurs.