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Perspectivity of Points in Matroid Lattices

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1. Introduction

In the theory of modular lattices, the perspectivity plays an important role. We have the theorem of perspective mappings, the comparability theorem and the theorem of distributivity and perspectivity (cf. [6] Theorems (1.1), (1.2) and (1.3)). But for the non-modular lattices, the properties of perspectivity are unknown.

U. Sasaki and S. Fujiwara [12] proved that in the matroid lattices the perspectivity of points is transitive. In this paper, starting from this significant fact, I investigate some properties of perspectivity in matroid lattices, and obtain the theorem of distributivity and perspectivity ((4.9) below) and the comparability theorem ((5.4) below). But the theorem of perspective mappings is as yet unsolved, even if we use the symmetric perspectivity (cf. (2.12) below).

In this paper, I treat the matroid lattices from the standpoint of atomistic symmetric lattices.

2. Symmetric lattices and matroid lattices.

In this paper we deal with a given lattice L with 0.

Definition (2.1). Let $a, b \in L$. $(a, b)M$ means $(c \cup a) \cap b = c \cup (a \cap b)$ for every $c \leqq b$, and $a \perp b$ means $a \cap b = 0$ and $(a, b)M$. If $a \perp b$ implies $b \perp a$, then L is called a *symmetric lattice* (cf. [13] p. 495). If $(a, b)M$ implies $(b, a)M$, then L is called a *M-symmetric lattice* (cf. [14] p. 453). And if $a \cap b \neq 0$ implies $(a, b)M$, then we call L a *weakly modular lattice* (cf. [1] p. 68).

A lattice L is called *left complemented* if $a, b \in L$ implies the existence of b_1 such that

$$a \cup b = a \cup b_1, \qquad a \cap b_1 = 0, \qquad (b_1, a)M, \qquad b_1 \leqq b$$

(cf. [14] p. 453).

Lemma (2.2). *In a symmetric lattice L, the binary relation "$\perp$" satisfies*