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Note on Miller's Recurrence Algorithm

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1. Introduction

In this paper, we are concerned with a recurrence algorithm, originated by J. C. P. Miller [1][1], for computing a solution f_n of a second-order difference equation

$$y_{n-1}=a_n y_n+b_n y_{n+1} \qquad (b_n \neq 0;\ n=1, 2, \cdots), \tag{1.1}$$

in the case where (1.1) has a second solution g_n which ultimately grows much faster than f_n [6]. This algorithm is used for computing Bessel functions [1, 2, 4, 9], Legendre functions [8], repeated integrals of the error function [3], and so on.

Let $P_n(k)$ be defined by the formula

$$P_n(k-1)=a_k P_n(k)+b_k P_n(k+1) \qquad (k=n+1, n, \cdots, 1), \tag{1.2}$$

where

$$P_n(n)=1, \quad P_n(n+1)=0, \qquad P_n(n+2)=1/b_{n+1}. \tag{1.3}$$

Then Miller's algorithm is applied in the following two ways:

1°. when the normalizing condition

$$m_0 f_0+m_1 f_1+\cdots=c \qquad (c \neq 0) \tag{1.4}$$

is known, put

$$S_n(k)=\frac{c P_n(k)}{R_n} \qquad (k=0, 1, \cdots, n), \tag{1.5}$$

where

$$R_n=\sum_{j=0}^{n} m_j P_n(j). \tag{1.6}$$

2°. when f_0 is known and $f_0 \neq 0$, put

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1) Numbers in square brackets refer to the references listed at the end of this paper.