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On Decompositions of Riemannian Manifolds.

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Introduction. In a homogeneous space G/H, each of its points is a coset of a closed subgroup H of a Lie group G, that is, G/H is based on the decomposition of G into the leaves (maximal integral manifolds) of the Lie algebra of H. Generalizing, from this point of view, the notion of a homogeneous space, we have that of a foliation $M/\mathfrak{M}$ in the sense of R. Palais [4], which consists of leaves of an involutive distribution $\mathfrak{M}$ on a differentiable manifold M, together with the topology induced from M (see p. 82). Foliations in more general spaces have been treated by C. Ehresmann [2], G. Reeb [5] and A. Haefliger [3].

In the present paper we shall investigate the decomposition of a Riemannian manifold M into the leaves of an involutive distribution $\mathfrak{M}$ which has the involutive orthogonal complement $\mathfrak{M}^*$. At first it will be tried to represent the foliation $M/\mathfrak{M}$ with a leaf V^* of $\mathfrak{M}^*$. This requires that the leaf should meet all the leaves of $\mathfrak{M}$. We shall find a sufficient condition of that in terms of certain quantities related to a family of geodesic curvatures (Theorem 2). Under this condition, for any simply connected leaf V^* of $\mathfrak{M}^*$, we have the relation:

$$M/\mathfrak{M} \cong V^*/G(V^*)$$

(Theorem 3), where $G(V^*)$ denotes the group of diffeomorphisms of V^* which make the intersection of V^* and each leaf of $\mathfrak{M}$ invariant. Let H_p be the subgroup of $G(V^*)$ consisting of elements which make a point p invariant. Then, if H_p and H_q are conjugate subgroups, it will be shown that the leaves of $\mathfrak{M}$ through p and q are diffeomorphic. And, if one of the leaves of $\mathfrak{M}$ is simply connected, if the leaves of $\mathfrak{M}$ through p and q are homeomorphic, then H_q and H_p are isomorphic (Theorem 4). Finally we shall show that, when $G(V^*)$ is abelian, its elements can be extended to diffeomorphisms of M which make the decomposition of M invariant (Theorem 5).

1. Let M be an n-dimensional differentiable[1)] manifold with countable base. Let us be given an m-dimensional involutive distribution $\mathfrak{M}$ on M. Since, as is well known, M can be given a Riemannian structure, we have, at any point

1) By "differentiable" we always mean "of class C^∞".