J. Sci. Hiroshima Univ. Ser. A-I
29 (1965), 11-15

Notes on the Theory of Differential Forms on Algebraic Varieties

Yoshikazu Nakai

(Received February 27, 1965)

This note contains two rather separate topics. The first theorem is a version of Lefschetz' theorem in the language of differential forms. The second one is a characterization of abelian subvariety of an abelian variety. They are a continuation of our preceding papers [2] and [3][1]. As addenda we shall give corrections to the cited papers [2] and [3].

§ 1. Isomorphism of j_Y^*.

We shall prove in this § the following

Theorem 1.1. *Let X^n be a non-singular projective variety and let Y be a non-singular irreducible hypersurface section of X of order m. Let j_Y be the injection $Y \rightarrow X$ and j_Y^* be its adjoint map $H^0(X, \Omega_X) \rightarrow H^0(Y, \Omega_Y)$, where Ω_X, Ω_Y are the sheaves of germs of regular differential forms of degree 1 on X and Y respectively. Then if $n \geqslant 3$ and m is sufficiently large j_Y^* is an isomorphism of $H^0(X, \Omega_X)$ and $H^0(Y, \Omega_Y)$.*

We have proved already in [2] that j_Y^* is an injective map provided m is sufficiently large (Theorem 5 in [2]). Hence to prove Theorem 1 it suffices to prove the following:

Proposition 1.2. *Let X be as in* Theorem 1 *and let $\mathcal{O}$ be the structure sheaf of X and let m_0 be an integer such that*

(1) $$H^i(X, \mathcal{O}_X(-m)) = 0 \ \textit{for} \ m \geqslant m_0 \ \textit{and} \ i = 1, 2.$$

(2) $$H^1(X, \Omega_X(-m)) = 0 \ \textit{for} \ m \geqslant m_0.$$

If Y is a generic hypersurface section of order $\geqslant m_0$, then j_Y^* is a surjective map.

Proof. Let us denote by $\mathscr{P}$ the sheaf of ideals defined by Y, i.e., the sheaf of germs of rational functions f such that $(f) > Y$. As before let Ω_X, Ω_Y be the sheaves of germs of regular differential forms on X and Y respectively.

1) The numbers in the bracket refer to the bibliography at the end of the paper.