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On the Multiplicative Products of x_+^α and x_+^β

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In my previous paper [3] we examined the relationship between the different approaches of defining multiplication between distributions. We consider only distributions defined on the real line R. The definition of multiplicative product due to Y. Hirata and H. Ogata [2] is equivalent to the one given by J. Mikusiński [4]. In the sequel the multiplicative product in this sense of two distributions S, T, if it exists, will be denoted by ST. We have shown in [6] that ST exists if and only if $(\phi S)*\check{T}$, $\phi \in \mathcal{D}$, when restricting it to a neighbourhood of 0, is a bounded function continuous at 0. Another approach suggested by H. G. Tillmann runs as follows: let $\hat{S}(z)$ and $\hat{T}(z)$ be locally analytic functions corresponding to S and T respectively ([7], p. 122). Putting $\hat{S}_\varepsilon(x)=\hat{S}(x+i\varepsilon)-\hat{S}(x-i\varepsilon)$ and $\hat{T}_\varepsilon(x)=\hat{T}(x+i\varepsilon)-\hat{T}(x-i\varepsilon)$, $\varepsilon>0$, he defined the product $S\cdot T$ to be $\lim\limits_{\varepsilon\to 0}\hat{S}_\varepsilon\hat{T}_\varepsilon$ if it exists, or more generally the finite part of $\hat{S}_\varepsilon\hat{T}_\varepsilon$ (in Hadamard's sense) if it exists. As in my previous paper [3], we understand by $S\bigcirc T$ the distributional limit $\lim\limits_{\varepsilon\to 0}\hat{S}_\varepsilon\hat{T}_\varepsilon$ if it exists. We have shown in [3] that if ST exists, then $S\bigcirc T$ exists and coincides with ST, but not conversely.

The main purpose of this paper is to make a comparison between the various multiplications indicated above when S and T are x_+^α and x_+^β respectively.

1. Preliminaries

It is shown in [6] that if $\dfrac{dS}{dx}T$ exists, then $S\dfrac{dT}{dx}$, ST exist and $\dfrac{d}{dx}(ST)=\dfrac{dS}{dx}T+S\dfrac{dT}{dx}$. Let $\mathcal{D}'_+$ be the set of all distributions with supports in the positive real axis.

Proposition 1. *Let Y be the Heaviside function. Let T be $\dfrac{dS}{dx}$. Then YT exists if and only if there exists a neighbourhood U of 0 in R such that S is a bounded function in U and is continuous at 0. When YT exists, $YT=\dfrac{d}{dx}(YS)-S(0)\delta$ and especially $YT=T$ for $T\in\mathcal{D}'$.*

Proof. Suppose YT exists. Then YS exists. In view of the relation: