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Composition of some Series of Association Algebras

Sumiyasu Yamamoto, Yoshio Fujii* and Noboru Hamada
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1. Introduction

The concept of association schemes was introduced first by Bose and Shimamoto [4]. It was investigated in relation to the definition of the partially balanced incomplete block (PBIB) designs introduced first by Bose and Nair [3]. This concept, however, has recently been treated without referring to the definition of the PBIB designs. An association scheme with m associate classes, which is defined among v objects, usually called treatments, is a relation of association defined among those satisfying the following three conditions:

(i) Any two treatments are either 1st, 2nd, $\cdots$, or m-th associates, the relation of association being symmetrical. Each treatment is the zeroth associate of itself.

(ii) Each treatment α has n_i i-th associates, the number n_i being independent of α.

(iii) If any two treatments α and β are i-th associates, then the number of treatments which are j-th associates of α and k-th associates of β is p^i_{jk} and is independent of the pair of i-th associates α and β.

Matrix representation of the relationship of association along the concept of relationship algebra by James [8] was immediately followed by the definition of the association algebra by Bose and Mesner [2]. The structure of the association algebras was studied by Ogawa [14], [15] in some detail. Further steps were taken by Yamamoto and Fujii [23].

An association algebra with m associate classes is a semi-simple commutative matrix algebra generated by the association matrices $A_0, A_1, \cdots, A_m$ over the real field. It is completely reducible and its minimum two sided ideals are linear. The principal idempotents $A^\#_0, A^\#_1, \cdots, A^\#_m$ of those ideals and the association matrices are mutually linked by the linear combinations of the others. That is, $A_i=\sum_{j=0}^{m} z_{ji}A^\#_j$ and $A^\#_i=\sum_{j=0}^{m} z^{ij}A_j$ where $z^{ij}=\alpha_i z_{ij}/vn_j$ and $\alpha_i=$ rank $(A^\#_i)$. In many cases it happens that all of the idempotent matrices are rational. In such cases we consider the association algebras to be defined over the rational field.

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*) Present address: Department of Mathematics, Faculty of Science, Kanazawa University, Kanazawa, Japan. This work was supported in part by a research grant of the Sakkokai Foundation.