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Semi-modular Lie Algebras

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Introduction

If L is a Lie algebra over a field Φ, we consider the lattice of all subalgebras of L. A Lie algebra will be called distributive, modular, upper semi-modular or lower semi-modular if its lattice of subalgebras has the corresponding property. In this paper we investigate the relation between the structure of the Lie algebra and the structure of its lattice of subalgebras. Analogous work has been done for groups by many investigators, and Suzuki [7] has written a comprehensive monograph describing the significant results in this area.

The Lie algebras considered in this paper will be finite dimensional. Also, the Lie algebras will, unless otherwise stated, be over a field of characteristic zero.

In this paper, if L is a Lie algebra $[L, L]$ will be denoted by L', and $[L', L']$ by L''. Also the subalgebra of L generated by $e_1, e_2, \cdots, e_k$ will be denoted by $\{e_1, e_2, \cdots, e_k\}$.

In this paper, we 1) characterize upper semi-modular Lie algebras over fields of characteristic zero, 2) characterize modular Lie algebras over fields of characteristic zero, 3) characterize lower semi-modular Lie algebras over algebraically closed fields of characteristic zero, 4) study other properties of distributive, modular, upper semi-modular, lower semi-modular Lie algebras.

1. Preliminaries and examples

Definition: A Lie algebra L over a field of any characteristic is called distributive, modular, upper semi-modular or lower semi-modular if its lattice of all subalgebras has the corresponding property.

If a Lie algebra is distributive, modular, upper semi-modular or lower semi-modular, then a subalgebra or a factor algebra has the corresponding property.

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