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*Chordal Limits of Holomorphic Functions at Plessner Points**)

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The purpose of this paper is to construct an example of a holomorphic function in the open unit disk D of the complex plane that has a certain kind of boundary behavior at every point of the unit circle Γ. Before describing our example, we introduce some notation and terminology, and then discuss some results of Kurt Meier to which our work is closely related, in order to place it in its proper setting.

Let f be a meromorphic function whose domain is D and whose range is a subset of the Riemann sphere Ω. We assume that the reader is familiar with some of the elementary notions of cluster set theory (see [5]). Thus, the cluster set of f at a point $\zeta \in \Gamma$ is denoted by $C(f, \zeta)$. If X is a chord at ζ, then $C_X(f, \zeta)$ denotes the corresponding chordal cluster set of f at ζ. We say that f has a chordal limit at ζ provided that there exists a chord X at ζ and a value $\omega \in \Omega$ such that $C_X(f, \zeta)=\omega$; if, in particular, X is the radius at ζ, then ω is called the radial limit of f at ζ. We suppose that the reader knows what is meant when we say that f has an angular limit at a point $\zeta \in \Gamma$.

We define the chordal principal cluster set of f at a point $\zeta \in \Gamma$ as the set

$$\Pi_X(f, \zeta)\equiv\bigcap_X C_X(f, \zeta),$$

where X ranges over the set of all chords at ζ. The angular range of f at ζ is defined to be the set $\Lambda(f, \zeta)$ of all values $\omega \in \Omega$ with the property that f assumes the value ω in every Stolz angle at ζ arbitrarily close to ζ.

We also take for granted that the reader knows what is meant by a Fatou point of f and by a Plessner point of f. A Meier point of f is defined to be a point $\zeta \in \Gamma$ at which

$$\Pi_X(f, \zeta)=C(f, \zeta)\subset\Omega,$$

where the symbol $\subset$ signifies proper inclusion. By an angular Picard point of f we mean a point $\zeta \in \Gamma$ at which the set $\Omega-\Lambda(f, \zeta)$ contains at most two values. Finally, we define an alternative point of f to be a point $\zeta \in \Gamma$ at which

$$\Pi_X(f, \zeta)\cup\Lambda(f, \zeta)=\Omega.$$

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