

On the Value of Distributions at a Point and the Multiplicative Products

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The theory of multiplication between distributions has been developed by several authors (cf. the references in [5]). Recently M. Itano [5] defined the multiplication satisfying certain reasonable requirements. Such a multiplication was called normal by him. In his theory the notion of a value of a distribution at a point in the sense of S. Łojasiewicz [7] plays an important rôle. On the other hand, in our previous paper [11] the multiplication was defined by using the δ -sequences. The aim of the present paper is to unify these two approaches of defining multiplication. To this end we shall introduce the notion of a δ -sequence in a restricted sense in order to make clear the relationships among different approaches to the theory of multiplication between distributions.

Let T be a distribution defined on R^N . If $\lim_{n \rightarrow \infty} \langle T, \rho_n \rangle$ exists for every δ -sequence $\{\rho_n\}$, T is equivalent to a function bounded in a 0-neighbourhood and continuous at 0 [11, p. 229]. Therefore, to reach the notion of the value of a distribution at a point, it will be necessary to restrict the δ -sequence. This is done in Section 1, where besides the value in the sense of Łojasiewicz we investigate the value introduced by Itano [5, p. 174] in one-dimensional case. Section 2 deals with the multiplicative product defined by Itano. On the model of the methods discussed in our previous paper [11], several equivalent conditions for the existence of the product are established. The various approaches of defining multiplication between distributions are also discussed. In Section 3 we introduce the notion on a $\delta_{\mathcal{H}}$ -sequence consisting of functions which belong to a normal space \mathcal{H} of distributions and may have non-compact supports. In virtue of this notion we can generalize the method of defining the multiplication due to H. J. Bremermann and L. Durand [2], H. G. Tillmann [13] and others. These multiplications are not normal in the sense of Itano [5]. Most results in the preceding Section 2 are carried over to these cases by replacing a restricted δ -sequence by $\delta_{\mathcal{H}}$ -sequence. The final Section 4 is devoted to presenting some examples concerning the values and multiplications.

§1. The value of a distribution at a point

Let R^N be an N -dimensional Euclidean space. Points of R^N are denoted