J. Sci. Hiroshima Univ. Ser. A-I
31 (1967), 211-219

The Linear Hypotheses and Constraints

Sumiyasu Yamamoto and Yasunori Fujikoshi

(Received September 20, 1967)

1. Introduction and Summary

The purpose of this paper is to give a unified treatment of the classical least squares theory in the linear hypothesis model. The linear hypothesis model treated here can be summarized as

$$\boldsymbol{y}=\boldsymbol{\theta}+\boldsymbol{e} \tag{1.1}$$

where $\boldsymbol{y}$ is an $n\times 1$ vector of observations, $\boldsymbol{\theta}$ is the expectation of $\boldsymbol{y}$ which is known to belong to a specified linear subspace

$$\Xi=\{\boldsymbol{\theta}\,|\,\boldsymbol{\theta}=A\boldsymbol{\tau},\quad B\boldsymbol{\tau}=0\} \tag{1.2}$$

of the n dimensional Euclidean space E_n, and $\boldsymbol{e}$ is an $n\times 1$ vector of random errors which has the multivariate normal distribution with mean 0 and covariance matrix $\sigma^2 I_n$, σ^2 times the unit matrix I_n.

It is worthwhile to note that in our unified treatment no restriction is imposed on the known $n\times m$ matrix A and the known $l\times m$ matrix B. The matrix A may be called a design matrix. The matrix equation $B\boldsymbol{\tau}=0$ is a set of constraints imposed on the parameter vector $\boldsymbol{\tau}$. $B\boldsymbol{\tau}=0$ is in some cases a set of identifiability constraints of the parameter vector $\boldsymbol{\tau}$, a set of hypotheses to be tested and a set of more complex constraints. The matrices A and B and the parameter vector $\boldsymbol{\tau}$ jointly specify the linear subspace Ξ of E_n.

The least squares estimate of the parameter $\boldsymbol{\tau}$ in the extended sense and the projection operator to the space Ξ obtained by using the generalized inverse matrices are given in the Theorem of section 2. Some properties of the generalized inverse matrices and the projection operators are also given in section 2.

Our general formula given in the Theorem contains as its special cases the following three cases (i), (ii) and (iii):

(i) $\operatorname{rank}(A)=m$,

(ii) $\operatorname{rank}(A)<m$ and $\mathfrak{R}[A']\supset\mathfrak{R}[B']$,

(iii) $\operatorname{rank}(A)<m$ and $\mathfrak{R}[A']\cap\mathfrak{R}[B']=\{0\}$,

where X' denotes the transposed matrix of X and $\mathfrak{R}[X]$ denotes a vector space spanned by the column vectors of a matrix X.

The case (i) is the simplest. The case (ii) has been treated in connection with the theory of testing testable hypotheses by many authors (c.f. Goldman