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# *Direct Solution of Partial Difference Equations for a Rectangle*

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## 1. Introduction

In this paper, we are concerned with the direct solution of the systems of linear algebraic equations arising from the discretization of linear partial differential equations over a rectangle. Such a system is usually solved by means of the iterative methods, and the direct methods are rarely used because of storage capacity [11][1]. Among the direct methods, however, there are known the square root method [11], the hypermatrix method [9, 36], the tensor product method [18], the method of summary representation [32], the method of lines [12, 20, 25, 26, 27, 37, 46], and so on [13, 16, 23, 39, 40, 45].

Although the results stated in this paper are not all new, they are summarized in a somewhat unified form. The methods can easily be extended to the problems in higher dimensions and to the domains consisting of rectangles. Several examples to which the direct methods are applicable are presented.

## 2. Preliminaries

### 2.1 Tridiagonal matrices

Let $x$ be a real number and let $U_r(x)$ and $V_r(x)$ be the solutions of the difference equation

$$y_{r+1} - x y_r + y_{r-1} = 0 \qquad (r=0, 1, \cdots) \tag{2.1}$$

satisfying the initial conditions $y_{-1}=0$, $y_0=1$ and $y_{-1}=1$, $y_0=x/2$ respectively. Then, as is easily checked, we have the following

Lemma 1. *$U_r(x)$ and $V_r(x)$ are expressed as follows:*

$$U_r(x) = \begin{cases} \dfrac{\sinh(r+1)\omega}{\sinh\omega}, & 2\cosh\omega = x \qquad (x \geqq 2) \\[2ex] \dfrac{\sin(r+1)\theta}{\sin\theta}, & 2\cos\theta = x \qquad (|x| < 2) \\[2ex] (-1)^r \dfrac{\sinh(r+1)\omega}{\sinh\omega}, & 2\cosh\omega = |x| \quad (x \leqq -2) \end{cases}$$

1) Numbers in square brackets refer to the references listed at the end of this paper.