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On the Vector Bundles $m\xi_n$ over Real Projective Spaces

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§1. Introduction

Let ξ_n be the canonical line bundle over n-dimensional real projective space RP^n, and $m\xi_n$ the Whitney sum of m-copies of it.

The purpose of this note is to study the number *span* $m\xi_n$ of the linearly independent cross-sections of $m\xi_n$. These are related to the immersion problems of RP^n in the Euclidean space R^m by [2], and also to the submersion problems of $P_k^n = RP^n - RP^{k-1}$ in R^m by [7] and Theorem 2.4 below.

In §2, we study the simple properties of *span* $m\xi_n$. In order to make further calculations, we consider in §3 the Postnikov resolution of the universal sphere bundle and characterize the third k-invariant by the methods of [9], where the results obtained may be contained in [5]. These are applied to *span* $m\xi_n$ in §4, and we consider the submersion problems of P_k^n in §5. The author expresses his hearty thanks to Prof. M. Sugawara and Dr. T. Kobayashi for their valuable suggestions and discussions.

§2. Some properties of $m\xi_n$

If ξ is a real vector bundle, we denote by *span* ξ the maximum number of the linearly independent cross-sections of ξ. Especially, when M is a C^∞-manifold, we denote by *span* M the *span* $\tau(M)$, where $\tau(M)$ is the tangent vector bundle of M.

The following two lemmas are well known.

Lemma 2.1. *Let $f: X \to Y$ be a homotopy equivalence between CW-complexes X and Y, and ξ be a real vector bundle over Y. Then*

$$span\, f^{\#}\xi = span\, \xi,$$

where $f^{\#}\xi$ is the induced bundle of ξ by f.

Lemma 2.2. *Let ξ be a real vector bundle over a CW-complex X. If* $\dim \xi > \dim X$, *then* $span\, \xi \geqq \dim \xi - \dim X$, *and*

$$span(\xi \oplus 1) = 1 + span\, \xi,$$

where $\oplus$ is the Whitney sum and 1 in the left hand side is the 1-dimensional trivial bundle over X.

Now, let ξ_n be the canonical line bundle over the n-dimensional real pro-