J. Sci. Hiroshima Univ. Ser. A-I
32 (1968) 397-440

On the Distributional Boundary Values of Vector-Valued Holomorphic Functions

Mitsuyuki Itano

(Received September 26, 1968)

The representations of distributions as distributional boundary values of holomorphic functions have been discussed by many authors. As for a distribution on a unit circle, G. Köthe [10] developed the theory of holomorphic representations through the Cauchy integral along the unit circle of the given distribution. On the other hand, for a distribution on the real axis, H. G. Tillmann [25] constructed its holomorphic representation by making use of a technique similar to the method of G. Mittag-Leffler. There the representation is not unique and we may add an entire function to obtain another representing function. Recently, in his article [12] motivated by the works of M. Sato [16, 17], A. Martineau has developed the theory of the distributional boundary values of holomorphic functions, and reduced the problem of representing a distribution to solving a non-homogeneous Cauchy-Riemann equation. The solution is a distribution which is holomorphic except on the line where the original distribution is given.

For a vector-valued distribution on the real axis, H. G. Tillmann [26] has shown that the Cauchy integral along the real axis gives rise to a holomorphic representation if the distribution happens to be of almost compact support, while the method of G. Mittag-Leffler just referred to cannot be applied to an arbitrary vector-valued distribution. We are naturally led to the question whether it is possible to give holomorphic representations of an arbitrary vector-valued distribution on the real axis. We can show that the answer is negative. This is because the space of entire functions has no topological supplement in the space of the solutions of the Cauchy-Riemann equations mentioned above. On the contrary, any vector-valued distribution on a unit circle is holomorphically represented by the Cauchy integral.

The main purpose of the present paper is to develop a general theory on holomorphic representations of the vector-valued distributions so that we may be able to answer the question raised above even in a Riemann surface M. Here a vector-valued distribution is given on a real analytic 1-dimensional oriented closed submanifold Γ which need not be connected. The problem will be reduced to solving a non-homogeneous Cauchy-Riemann equation in accordance with the idea of A. Martineau. Naturally his method requires a modification in its treatment of the subject.

Special attention will be paid to the cases where the original distribution is given on the real axis or on a unit circle.