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Non-Immersion Theorems for Lens Spaces. II

Dedicated to Professor Atuo Komatu on his 60th birthday

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§1. Introduction

Throughout this note we assume that p is an odd prime. Let Z_p be the cyclic group of order p with generator γ. Let S^{2n+1} be the unit sphere in complex $(n+1)$-space. Define an action of Z_p on S^{2n+1} by the formula:

$$\gamma(z_0, z_1, \cdots, z_n) = (\lambda z_0, \lambda z_1, \cdots, \lambda z_n), \quad \text{where} \quad \lambda = e^{2\pi i/p},$$

for $(z_0, z_1, \cdots, z_n) \in S^{2n+1}$. The orbit space S^{2n+1}/Z_p is the lens space mod p and is written by $L^n(p)$. It is a compact, connected, orientable C^∞-manifold of dimension $2n+1$ and has the structure of a CW-complex with one cell in each dimension $0, 1, \cdots, 2n+1$. Let $L_0^n(p)$ be the $2n$-skeleton of $L^n(p)$.

The purpose of this paper is to prove some results on the stable homotopy type of the stunted space $L_0^n(p)/L_0^m(p)$ $(n>m)$ and on the non-immersibility of the lens space $L^n(p)$ in the Euclidean space.

After some preparations in §2, we determine the structure of the reduced Grothendieck ring $\tilde{K}(L_0^n(p)/L_0^m(p))$ of complex vector bundles in §3. Using the Adams operation we shall prove the following result in §4.

Theorem A. *Let $n>m$. If $L_0^n(p)/L_0^m(p)$ is stably homotopy equivalent to $L_0^{n+t}(p)/L_0^{m+t}(p)$, then $t \equiv 0 \pmod{p^{[(n-m-1)/(p-1)]}}$.*

We notice that the following result is known by Theorem 3 of [4]: $L_0^n(p)/L_0^m(p)$ is stably homotopy equivalent to $L_0^{n+t}(p)/L_0^{m+t}(p)$, if $t \equiv 0$ $(\bmod\, p^{[(n-m)/(p-1)]})$.

Together with Theorem 3 of [5], Theorem A can be used to give a condition for the immersibility of $L^n(p)$ in the Euclidean space $R^{2n+2m+1}$.

Theorem B. *Let n and m be integers with $n>m>0$. Assume that $n+m+1 \not\equiv 0 \pmod{p^{[(n-m-1)/(p-1)]}}$. If there is an immersion of $L^n(p)$ in $R^{2n+2m+1}$, then the Euler class of its normal bundle is zero.*

This will be proved in §5. From Theorem B we have the following result.

Theorem C. *Let n and m be integers with $n>m>0$. Assume that the following two conditions are satisfied:*