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On Certain Classes of Malcev Algebras

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This note is a sequel to the author's earlier paper [5]. For brevity we adopt the notations and definitions employed in [5] without explaining them here again. This note is concerned only with Malcev algebras (finite-dimensional) belonging to the classes of general algebras dealt with in [5]. As is well-known (see e.g. [6]), a *Malcev algebra* A is an anticommutative algebra satisfying the identity (x, y, z in A):

$$xy\cdot xz=(xy\cdot z)x+(yz\cdot x)x+(zx\cdot x)y.$$

In [5] we proved: *A solvable ideal B of an (A_3')-algebra A, whose derived series consists of ideals, is contained in the annihilator ideal of A.* For Lie algebras, which are a priori Malcev algebras, the derived series consists of ideals and the above result reduces to a known result as generalized by us earlier (see [5, Corollary 2.9]): *A solvable ideal B of a Lie (A_3)-algebra A is contained in the center of A.* On the other hand, for a Malcev algebra, the derived series need not in general consist of ideals (see [6, Example 3.4] for an example of such a Malcev algebra). However, we show (Proposition 1) that the above result for Lie algebras can be extended to Malcev algebras over fields of characteristic $\neq 2$, by using a recent result of Kuz′min ([2, Lemma 2]). The main interest in this extension lies in its application to the proof of a characterizing result (Theorem 4) for Malcev (A_k)-algebras ($k\geqslant 3$). This result considered along with some known results leads to an interesting conclusion (Theorem 5): *A Malcev (A_3)-algebra over an algebraically closed field of characteristic zero is abelian.*

In what follows, unless otherwise stated, A is a Malcev algebra over a field of characteristic $\neq 2$.

1. If B is an ideal of A, we write $B^{[1]}=B$, $B^{[2]}=BB+(BB)A$, $\cdots$, $B^{[n]}=B^{[n-1]}B^{[n-1]}+(B^{[n-1]}B^{[n-1]})A$, $\cdots$. $B^{[k]}$ are ideals of A. B is said to be *L-solvable* (see [2]) if there exists an integer n such that $B^{[n]}=0$. For such of those Malcev algebras as we here consider, L-solvability of an ideal is equivalent to its solvability in the usual sense. It is this result of Kuz′min ([2, Lemma 2]) which we employ for proving

Proposition 1. *A solvable ideal B of a Malcev (A_3)-algebra A is contained in the center of A.*