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On Certain Classes of Algebras

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Sugiura [4] and Jôichi [2] have studied the classes (A), (A_k), $k \geqslant 2$ and (A_∞) of Lie algebras. In this paper we define these as well as some other classes for general nonassociative algebras and obtain a characterization of alternative algebras over a field of characteristic zero belonging to any one of these classes. Incidentally, we obtain certain results which include striking improvements of earlier ones due to Sugiura (loc. cit.) and Jôichi (loc. cit.).

1. In what follows, A is a finite-dimensional nonassociative algebra over a field F and $L_x(R_x)$ denotes the left (right) multiplication by x in A. The classes mentioned at the outset are defined as follows:

DEFINITION 1.1. A is said to be an (X)-algebra, where (X) is any one of (A), (A'), $(A_k)_{k\geqslant 2}$, $(A'_k)_{k\geqslant 2}$, $(B_k)_{k\geqslant 2}$, $(B'_k)_{k\geqslant 2}$, (A_∞), (A'_∞), (B_∞), (B'_∞), according as the corresponding property (X) given below is satisfied (x, y, z, t in A):

(A) : $x(xy)=0 \Rightarrow xy=0$ and $(zt)t=0 \Rightarrow zt=0$

(A') : $x(xy)=0=(yx)x \Rightarrow xy=0=yx$

$(A_k)_{k\geqslant 2}$: $L_x^k=0 \Rightarrow L_x=0$ and $R_y^k=0 \Rightarrow R_y=0$

$(A'_k)_{k\geqslant 2}$: $L_x^k=0=R_x^k \Rightarrow L_x=0=R_x$

$(B_k)_{k\geqslant 2}$: $yL_x^k=0 \Rightarrow yL_x=0$ and $zR_t^k=0 \Rightarrow zR_t=0$

$(B'_k)_{k\geqslant 2}$: $yL_x^k=0=yR_x^k \Rightarrow yL_x=0=yR_x$

(X_∞) : (X) holds for all $k \geqslant 2$, $(X)=(A_k)$, (A'_k), (B_k), or (B'_k).

REMARK 1. The properties (A), (A_k), stated above may be further weakened by considering just the left or right multiplications independently. (This weakening has, of course, no significance for commutative or anticommutative algebras.) The resulting concepts of right (A)-algebra, left (A)-algebra, and (A)-algebra would then be distinct, (e.g.), the algebra A with basis u, v: $u^2=v^2=vu=0$; $uv=u$, is a right (A)-(also right (A_2)-) algebra but not a left (A)-(or left (A_2)-) algebra, i.e., not an (A)-(or (A_2)-) algebra.

The following chart indicates the connections among the properties mentioned in Definition 1.1.