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Multiplication Rings Containing Only Finitely Many Minimal Prime Ideals

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1. Introduction

A commutative ring R is called an *AM-ring* if whenever A and B are ideals of R with A properly contained in B, then there is an ideal C of R such that $A=BC$. An AM-ring in which $RA=A$ for each ideal A of R is called a *multiplication ring.* This paper is principally concerned with the results of a paper by Gilmer and Mott [7] when the ring R is assumed to contain only a finite number of minimal prime ideals. One of the principal results of this paper is that a multiplication ring R is Noetherian if and only if R contains only finitely many minimal prime ideals. Unless otherwise stated, all rings considered in this paper are assumed to be commutative and to contain an identity. However, on some occasions it will be pointed out that the theorem proved can be proved when R does not necessarily contain an identity.

2. Preliminary results and definitions

Two very important properties to be considered are the properties that will be called (*) and (**) throughout this paper. A ring R satisfies (*) if *an ideal of R with prime radical is primary*, and (**) is the property that *an ideal of R with prime radical is a prime power.* Also important is the notion of the *kernel* of an ideal, which is defined as follows: if $\{P_\alpha\}$ is the collection of all minimal prime ideals of an ideal A of R, then by an isolated P_α-primary component of A we mean the intersection Q_α of all P_α-primary ideals which contain A. The kernel of A is the intersection of all Q_α*'s.*

The relationship between properties (*) and (**) and the kernel of an ideal were studied in [7]. We list here those results which are used most frequently in this paper.

Theorem 1. *A ring R satisfies (*) if and only if R is one of the following:*

a) *a zero-dimensional ring,*

or

b) *a one-dimensional ring in which each non-maximal prime ideal P of R has the property that if M is a maximal ideal such that $P<M<R$ and if $p\in P$, then $p\in pM$.*

In Theorem 1, b) is equivalent to c) *R is one-dimensional and if P and M*