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A Remark on the Homeomorphism Group of a Manifold

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Let M be a connected separable metric space each point of which has a neighborhood (open set in the metric topology on M) whose closure in M is homeomorphic to $C_n(0;1)=\{x \in R^n \mid d(x, 0)\leqq 1\}$. Such a space M is simply called an *n-manifold*. Let $G(M)$ denote the group of all homeomorphisms of an n-manifold M, and $G^0(M)$ the subgroup of $G(M)$ generated by all h in $G(M)$ such that, for some *internal* closed n-cell F, $h|M-F=$ identity. Let $G^I(M)$ denote the subgroup of $G(M)$ generated by all h in $G(M)$ such that, for some closed n-cell F, $h|F=$ identity. Let $P(F)$ be the set of all h in $G(M)$ such that, for some f in $G^0(M)$, $h|F=f|F$, and $Q(F)$ the set of all h in $G(M)$ such that, for some f in $G^0(M)$, $h(F)=f(F)$ and $f^{-1}h|B$ is in $G^0(B)$, where F is an *internal* closed n-cell in M and $B=$ Bndy F. Let $T(F)$ be the set of all cells in M tame with respect to F.

It has been proved in [1] that, for an n-manifold $M(n\leqq 3)$, there is an internal closed n-cell F_0 in M such that, for any h in $G(M)$, there is an f in $G^0(M)$ such that $f(F_0)=h(F_0)$ (setwise), and then F_0 is called a *pivot* cell. In connection with such a pivot cell the following theorems have been obtained in [1].

Theorem 11 (Fisher). *Let M be a manifold,* dim $M\leqq 3$, *and let F_0 be a pivot cell in M. For every F in $T(F_0)$, $P(F)$ is a normal subgroup of $G(M)$. For each F in $T(F_0)$, $P(F)=P(F_0)$.*

Theorem 12 (Fisher). *Let M be a manifold,* dim $M\leqq 3$, *and let F_0 be a pivot cell in M. For any F in $T(F_0)$, $P(F)=G^I(M)$. Hence an h in $G(M)$ is in $G^I(M)$, so that $h=h_1\cdots h_k$, h_i the identity inside some closed n-cell F_i in M ($n=$* dim M), *if and only if for any n-cell F in M tame with respect to F_0, there is a deformation f of M such that $f(x)=h(x)$ for every x in F.*

Although the existence of the pivot cell in M is unknown for dim $M\geqq 4$, in this note we shall show that, for any internal closed n-cell F in an n-manifold $M(1\leqq n<\infty)$, the above theorems can be generalized as the following theorem and its corollary.

Theorem. *Let M be an n-manifold, and let F_0 and F be any internal closed n-cells in M. Then $P(F_0)=P(F)$ and $P(F)$ is a normal subgroup of $G(M)$.*

Proof. According to the theorem 1 in [1], there is an f in $G^0(M)$ such