J. Sci. Hiroshima Univ. Ser. A-I
33 (1969), 231-235

Comparison of the Classes of Wiener Functions

Fumi-Yuki Maeda

(Received September 20, 1969)

Introduction

For a harmonic space satisfying the axioms of M. Brelot [1], one can define the notion of Wiener functions as a generalization of that for a Riemann surface or a Green space (see [2]). The class of Wiener functions may be used to see global properties of the harmonic space; in particular, in order to show that a compactification of the base space be resolutive with respect to the Dirichlet problem, it is enough to verify that every continuous function on the compactification is a Wiener function (see Theorem 4.4 in [2]). Thus, given two harmonic structures $\mathfrak{H}_1$ and $\mathfrak{H}_2$ on the same base space Ω, it may be useful to know when the inclusion $\boldsymbol{BW}^{(1)} \subset \boldsymbol{BW}^{(2)}$ holds, where $\boldsymbol{BW}^{(i)}$ $(i=1, 2)$ is the class of bounded Wiener functions with respect to $\mathfrak{H}_i$ $(i=1, 2)$. In this paper, we shall give a sufficient condition for the above inclusion, which includes the conditions given in [4] and [5] for special cases.

1. Harmonic spaces and Wiener functions

In this paper, we assume that a harmonic space $(\Omega, \mathfrak{H}) = \{\mathfrak{H}(G)\}_{G:\text{open}}$, satisfies Axioms 1, 2 and 3 of M. Brelot [1] and that Ω is non-compact. For an open set G in Ω, the set of all superharmonic functions on G with respect to $(\Omega, \mathfrak{H})$ is denoted by $\mathcal{S}_{\mathfrak{H}}(G)$. The set of all potentials with respect to $(\Omega, \mathfrak{H})$ is denoted by $\mathcal{P}_{\mathfrak{H}}$. In general, given a family $\boldsymbol{A}$ of (extended) real-valued functions, we use the notation $\boldsymbol{A}^+ = \{f \in \boldsymbol{A};\, f \geqq 0\}$ and $\boldsymbol{BA} = \{f \in \boldsymbol{A};\, f\text{: bounded}\}$.

We furthermore assume that $(\Omega, \mathfrak{H})$ satisfies

Axiom 4. $\quad 1 \in \mathcal{S}_{\mathfrak{H}}(\Omega)$ *and* $\mathcal{P}_{\mathfrak{H}} \neq \{0\}$.

Remark that under Axiom 4 the following minimum principle holds (see [1]):

If $v \in \mathcal{S}_{\mathfrak{H}}(\Omega)$ and if for any $\varepsilon > 0$ there exists a compact set K in Ω such that $v(x) > -\varepsilon$ on $\Omega - K$, then $v \geqq 0$.

Given an extended real-valued function f on Ω, we consider the classes

$$\overline{\mathcal{W}}_{\mathfrak{H}}(f) = \left\{ v \in \mathcal{S}_{\mathfrak{H}}(\Omega);\ \begin{array}{l}\text{there exists a compact set } K_v \text{ in } \Omega \\ \text{such that } v \geqq f \text{ on } \Omega - K_v\end{array} \right\}$$

and