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Some Properties of the Kuramochi Boundary

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Introduction

It has been shown that the Kuramochi boundary of a Riemann surface or of a Green space has many useful potential-theoretic properties (see [9], [4], [11], etc.). In this paper, we shall give a few more properties of the Kuramochi boundary.

We consider a Green space Ω in the sense of Brelot-Choquet [3] and denote by Ω^* its Kuramochi compactification of Ω (see [4], [9] and [14] for the definition). Let Γ be the harmonic boundary on $\Delta=\Omega^*-\Omega$, i.e., the support of a harmonic measure $\omega\equiv\omega_{x_0}$ ($x_0 \in \Omega$). By definition, Γ is a non-empty closed subset of Δ.

Let K_0 be a fixed compact ball in Ω. For any resolutive function φ on Δ, let H_φ be the Dirichlet solution on $\Omega-K_0$ with boundary values φ on Δ and 0 on ∂K_0 (=the relative boundary of K_0). For the existence of H_φ, see e.g. [11]. If φ is a function on Γ and is the restriction of a resolutive function $\tilde{\varphi}$ on Δ, then $H_{\tilde{\varphi}}$ is uniquely determined by φ; we denote it also by H_φ. With this convention, we consider the space $\boldsymbol{R}_D(\Gamma)$ of functions φ on Γ which are restrictions of resolutive functions on Δ and for which $H_\varphi \in \boldsymbol{HD}_0$. Here, $\boldsymbol{HD}_0$ is the space of all harmonic functions u on $\Omega-K_0$ having finite Dirichlet integral $D[u]$ on $\Omega-K_0$ and vanishing on ∂K_0. Identifying two functions which are equal ω-almost everywhere, we can define a norm $\|\cdot\|$ on $\boldsymbol{R}_D(\Gamma)$ by

$$\|\varphi\|^2=D[H_\varphi]$$

for $\varphi \in \boldsymbol{R}_D(\Gamma)$.

In this paper, we shall show the following three properties: (1) The space $\boldsymbol{R}_D(\Gamma)$ is a Dirichlet space in the sense of Beurling-Deny [1] on Γ; (2) The capacity on Γ associated with this Dirichlet space coincides with the Kuramochi capacity ([9] and [4]); (3) The solution of a boundary value problem (of Neumann type) is expressed in terms of the Kuramochi kernel.

1. Dirichlet space $\boldsymbol{R}_D(\Gamma)$

The following lemma is a consequence of Lemma 5.3 in [13] (also cf. [11]):

LEMMA 1. *There exists a constant $M>0$ such that*