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# *Duality Theorems for Continuous Linear Programming Problems*

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## § 1. Introduction

Continuous linear programmings were first considered by W.F. Tyndall [7] as a generalization of "bottle-neck problems" in dynamic programming. N. Levinson [6], M.A. Hanson [3] and M.A. Hanson and B. Mond [4] generalized the results in [7].

In this paper we shall apply the theory of infinite linear programming studied by K.S. Kretschmer [5] and M. Yamasaki [8] to the investigation of the continuous linear programmings. Our main purpose is to improve the duality theorems in [6] and [7] obtained by approximation from the classical finite duality theorem.

In order to state the continuous linear programmings, we shall introduce some notation. If $D(t)$ is a matrix on the interval $[0, T]$ $(0<T<\infty)$ in the real line with entries $d_{ij}(t)$ and $g(t)$ is a scalar on $[0, T]$ such that every entry satisfies

$$d_{ij}(t) \leqq g(t),$$

then the notation

$$D(t) \leqq g(t)$$

will be used. If $\tilde{D}(t)$ is a matrix on $[0, T]$ with the same number of rows and columns as $D(t)$, then $D(t) \leqq \tilde{D}(t)$ means that $d_{ij}(t) \leqq \tilde{d}_{ij}(t)$ for all entries. For a matrix $D=(d_{ij})$ and a vector $d=(d_i)$, we set

$$|D| = \sum_{i,j} |d_{ij}| \text{ and } |d| = \sum_i |d_i|.$$

For an $n$ vector $d$, an $m$ vector $e$ and an $n \times m$ matrix $D$, let $dD$ and $De$ denote the vector-matrix products. Note that we do not use the familiar notation $Dd^T$. For two $n$ vectors $x(t)=(x_i(t))$ and $y(t)=(y_i(t))$, we set

$$x(t) \cdot y(t) = \sum_{i=1}^{n} x_i(t) y_i(t).$$

In this paper we always assume that

$$B(t)=(b_{ij}(t)) \text{ is an } n \times m \text{ matrix on } [0, T],$$