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On Certain Classes of Algebras-II

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The present note owes its origin to some remarks by Professor K. McCrimmon on an earlier paper with the same title [5]. The main object of this note is to show that the restriction on the characteristic of the base field can be dispensed with in Theorem 3.2 (and therefore also in Theorem 3.3) of [5] and that it can be weakened considerably in Proposition 3.4 of [5]. In other words, we prove

A. (i) *An alternative algebra over a field F of arbitrary characteristic is an (A_2')-algebra iff it is a direct sum of a zero ideal, and ideals which are (alternative) division algebras over F.*

(ii) *For an alternative algebra over F, all the properties stated in* [5, Definition 1.1] *are mutually equivalent.*

B. *A Jordan algebra over a field K of characteristic $\neq 2$ is an (A_k)-algebra for $k \geqslant 3$, iff it is either a zero algebra or is the direct sum of its annihilator ideal and the semisimple ideal A^2 such that there exists no nonzero element x in A^2 with ${R_x}^k = 0$, for the right multiplication R_x in A.*

Other results proved in this note are in the nature of some further remarks on the classes (A_k) of algebras supplementing those in [5].

The notations of this note are those of [5], and we consider only vector spaces which are finite dimensional over their base fields.

1. The following two lemmas which lead to results A, B are essentially based on an idea suggested by Professor McCrimmon.

Lemma 1.1. *Let A be a power-associative algebra* (see [7, Chapter V] for definition etc.) *over a field F and N be any nilideal of A. Then, for any idempotent $\bar{x} = x + N$ of A/N, there exists an idempotent e in A such that $\bar{e} = \bar{x}$, where $x \to \bar{x}$ is the canonical homomorphism of A onto A/N.*

Proof. By power-associativity of A/N, since $\bar{x}$ is an idempotent of A/N, $\bar{x}^n = \bar{x}$ for any integer n. Consequently x cannot be nilpotent in A; the associative subalgebra $F[x]$ of A generated by x is nonnil; $F[x]$ contains an idempotent e [7, Proposition 3.3]. We have $e = \sum_{i=1}^{n} a_i x^i$ for a_i in F. $\bar{e} = (\sum a_i)\bar{x} = b\bar{x}$ (say) is an idempotent in A/N where b is a nonzero element of F (since e cannot belong to N). The relations $\bar{e}^2 = \bar{e}$, $\bar{x}^2 = \bar{x}$ immediately yield $b = 1$ and the lemma is proved.