

Asymptotic Expansions of the Distributions of Test Statistics in Multivariate Analysis

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0. Introduction. Non-null distributions of some statistics in multivariate analysis have been expressed by series of zonal polynomials due to James [19], especially in terms of hypergeometric functions of matrix argument (Herz [11], Constantine [6] and James [21]). Such examples have been summarized in James [21], [22]. However the exact distributions of many test statistics are not yet available for the general values of parameters, and the almost all results obtained in some special cases are very complicated. Therefore, the asymptotic approximations for the distributions are very important.