J. Sci. HIROSHIMA UNIV. SER. A-I
34 (1970), 69-72

On Purely Inseparable Extensions of Algebraic Function Fields

Kôtaro KOSAKI and Hiroshi YANAGIHARA

(Received March 30, 1970)

In this note we shall be concerned with modular purely inseparable extensions of algebraic function fields over a perfect field k of a positive charactristic p. We shall first see that such an extension has a close connection with separating transcendence bases (Proposition 1), and then give a geometric interpretation of it (Proposition 2). Then if α is a purely inseparable isogeny of a group variety G onto another one G' defined over k, we shall show that the rational function field $k(G)$ of G over k is a modular extension of $\alpha^*(k(G'))$ by using some results by P. Cartier and M. E. Sweedler in [2], [5] and [6], where α^* is the comorphism corresponding to α (Proposition 3), and from this fact we shall show the existence of a favourable system of local parameters at the unit point e of G with respect to α (Theorem and its Corollary).

1. In the sequel let k be a perfect field of a positive characteristic p exclusively.

LEMMA 1. *Let K be an algebraic function field over k and L a purely inseparable extension of exponent 1 over K such that $[L\colon K]=p^s$. Then there exists a separating transcendence basis $\{t_1,\cdots, t_n\}$ of L over k such that $L=K(t_1,\cdots, t_s)$ and that $\{t_1^p,\cdots, t_s^p, t_{s+1},\cdots, t_n\}$ is a separating transcendence basis of K over k.*

This result is contained in the proof of Barsotti's Theorem in §2. 3 of [1]. Therefore we omit the proof.

PROPOSITION 1. *Let K be an algebraic function field over k and L a purely inseparable extension of K such that L is isomorphic to a tensor product $K(x_1)\otimes_K\cdots\otimes K(x_s)$ of simple extensions $K(x_i)$ over K. Then the transcendental degree n is not less than s and there exist $n-s$ elements $t_{s+1},\cdots, t_n$ in K such that $\{x_1,\cdots, x_s, t_{s+1},\cdots, t_n\}$ (resp. $\{x_1^{p^{e_1}},\cdots, x_s^{p^{e_s}}, t_{s+1},\cdots, t_n\}$) is a separating transcendence basis of L over k (resp. K over k), where e_i is the exponent of x_i over K for $i=1, 2,\cdots, s$.*

PROOF. If we put $y_i=x_i^{p^{e_i-1}}$ for each $i=1, 2,\cdots, s$, $L'=K(y_1,\cdots, y_s)$ is isomorphic to $K(y_1)\otimes_K\cdots\otimes K(y_s)$ and is of exponent *1* over k. By Lemma 1, there exists a separating transcendence basis $\{t_1,\cdots, t_n\}$ of L' over k such that $\{t_1^p,\cdots, t_s^p, t_{s+1},\cdots, t_n\}$ is that of K over k. Then we can easily see that