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Note on the Span of Certain Manifolds

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§1. Introduction

For a real vector bundle ξ, we denote by *Span* ξ the maximum number of the linearly independent cross-sections of ξ. Especially, we denote *Span* $M = Span\ \tau M$, where τM is the tangent bundle of a C^∞-manifold M.

In this note, we prove the following theorem, which is the conjecture of D. Sjerve [4, p. 104, (4.6)].

Theorem 1. *Let π denote any finite group of odd order, not necessarily abelian, acting freely as diffeomorphisms on some standard sphere S^n, and $M^n = S^n/\pi$ be the orbit manifold. Then*

$$Span\ M^n = Span\ S^n$$

holds for $n \neq 7$.

Also, we shall give counter examples to the following conjecture of E. Thomas [7, p. 655, Conjecture 5] by $S^1 \times P_n(C)$ and the mod 3 standard lens space $L^3(3)$, where $n = u \cdot 2^{2+4d} - 1$ (u: odd, $d \geqq 1$) and $P_n(C)$ is the complex n-dimensional projective space.

Conjecture of E. Thomas: *Let M be a compact n-manifold, n odd, and let k be a positive integer such that $k \leqq Span\ S^n$. If $w_1 M = \cdots = w_k M = 0$, then $Span\ M \geqq k$, where $w_i M$ is the i-th Stiefel-Whitney class of M.*

§2. Proof of Theorem 1

Theorem 2. [5, p. 551], [6, p. 53]. *Let ξ^n be an orientable n-dimensional real vector bundle over an n-dimensional complex X. Then,*

$$Span\ \xi^n < Span S^n \textit{ implies } Span\ (\xi^n \oplus 1) = 1 + Span\ \xi^n,$$

where $\xi^n \oplus 1$ is the Whitney sum of ξ^n and 1-dimensional trivial bundle over X.

Proof. Put $k = \mathrm{Span}\ (\xi^n \oplus 1)$, then there exists an $(n+1-k)$-dimensional vector bundle η over X such that $\xi^n \oplus 1 = \eta \oplus (k-1) \oplus 1$. So, by [6, Theorem 1], *Span* $(\eta \oplus (k-1)) = Span\ \xi^n$. This implies $Span (\xi^n \oplus 1) \leqq 1 + Span\ \xi^n$. And, *Span* $(\xi^n \oplus 1) \geqq 1 + Span\ \xi^n$ is clear. *q.e.d.*

Next, we notice that the following theorem holds for the odd-dimensional manifold of Theorem 1. This theorem is Theorem A in [3, p. 545] where π