HIROSHIMA MATH. J.
1 (1971). 75-108

Energy Inequalities and Cauchy Problems for a System of Linear Partial Differential Equations

Mitsuyuki ITANO and Kiyoshi YOSHIDA

(Received February 20, 1971)

In the present paper we consider a system of linear partial differential operators with variable coefficients written in matrix notation

$$Lu = D_t^m u + \sum_{j=0}^{m-1} \sum_{j+|p|\leq m} A_{j,p}(t, x) D_t^j D_x^p u, \qquad (m \geqq 1)$$

where the $A_{j,p}(t, x)$ are $N \times N$ matrices whose entries lie in $\mathcal{B}(H)$, H being a slab $0 \leqq t \leqq T$, $x \in R_n$. By $u \in \mathcal{D}'(\mathring{H})$ we mean that each component of u lies in $\mathcal{D}'(\mathring{H})$. To simplify the notation, similar abbreviations will constantly be used for vector distributions. A Cauchy problem for L with $t=0$ as initial hyperplane has been formulated in a generalized sense in a related paper [6]: To find in $\mathcal{D}'(\mathring{H})$ a solution u satisfying

$$Lu = f \qquad \text{in } \mathring{H}$$

under the condition

$$u_0 \equiv \lim_{t \downarrow 0} (u, D_t u, \dots, D_t^{m-1} u) = \alpha$$

for preassigned $f \in \mathcal{D}'(\mathring{H})$ and $\alpha \in \mathcal{D}'(R_n)$. Here $\lim_{t \downarrow 0} u$ denotes the distributional boundary value of u which was defined in [6] in accord with S. Łojasiewicz [10]. If a solution u exists, f must have a canonical extension over $t=0$. If this is a case and u satisfies $Lu = f$ in $\mathring{H}$, then u_0 exists if and only if u has an extension over $t=0$, that is, u is a restriction to $\mathring{H}$ of a $U \in \mathcal{D}'((-\infty, T) \times R_n)$. Most spaces of distributions encountered in the usual treatments of partial differential equations have such an extension property. For example, as for $\mathcal{H}_{(\sigma,s)}(H)$, the property is involved in its definition [4].

The purpose of this paper is to investigate Cauchy problems for L from our stand-points, imposing on L or L^* some additional conditions such as energy inequalities of Friedrichs-Levy type. While we regard such inequalities as a priori estimates, they are usually deduced from the properties involved in a differential system called hyperbolic.

In Section 2 we deal with energy inequalities with the aid of the lemmas given in Section 1. The equivalence of $[E_{(0)}]$ and $[E_{(s)}]$ are shown. In Sec-