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Commutative Rings for which Each Proper Homomorphic Image is a Multiplication Ring. II

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This paper is an extension of Wood's results in [4]. All rings considered are assumed to be nonzero commutative rings. A ring R is called an *AM-ring* if whenever A and B are ideals of R with A properly contained in B, then there is an ideal C of R such that $A=BC$. An AM-ring in which $RA=A$ for each ideal A of R is called a multiplication ring. Wood characterized in [4] rings with identity for which *each proper homomorphic image is a multiplication ring.* Such rings are said to satisfy property (Hm). An example is given in [4] to show that a ring satisfying (Hm) need not be a multiplication ring. In fact, a general method is given for constructing such examples. This paper considers u-rings satisfying property (Hm) where a ring S is called a *u-ring* if the only ideal A of S such that $\sqrt{A}=S$ is S itself. Section 2 shows that the characterization of rings with identity satisfying (Hm) carries over to u-rings safisfying (Hm).

The notation and terminology is that of [5] with two exceptions: $\subseteq$ denotes containment and $\subset$ denotes proper containment, and we do not assume that a Noetherian ring contains an identity. If A is an ideal of a ring R, we say that A is a *proper ideal* of R if $(0)\subset A\subset R$ and that A is a *genuine* ideal of R if $A\subset R$.

1. Rings Satisfying Properties $(H*)$ and $(H**)$.

Let R be a ring. We say that R *satisfies property* $(*)$ (*satisfies property* $(**)$) if each ideal of R with prime radical is primary (is a prime power). If each proper homomorphic image of R satisfies $(*)$ (satisfies $(**)$), we say that R *satisfies porperty* (H^*) (*satisfies property* $(H**)$). In [3] it is shown that an AM-ring satisfies $(*)$ and $(**)$ and that if S is a u-ring, S satisfies $(**)$ if and only if S satisfies $(*)$ and primary ideals are prime powers. Therefore, in a u-ring, $(H**)$ implies $(H*)$. We give here a partial characterization of u-rings satisfying $(H*)$ and then show that the characterization of u-rings satisfying $(H**)$ is the same as the characterization of rings with identity satisfying $(H**)$.

DEFINITION. A ring R is said to have *dimension* n or to be *n-dimensional* if there exists a chain $P_0\subset P_1\subset\cdots\subset P_n$ of $n+1$ prime ideals of R where $P_n\subset R$, but no such chain of $n+2$ prime ideals exists in R.