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A Note on the Admissible Tests and Classifications in Multivariate Analysis

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0. Summary

Kiefer and Schwartz [1] provided a general method of proving admissibility of tests in normal multivariate analysis. Using their method, in this paper, we prove admissibility of certain test procedures for the equality of a covariance matrix to a given one in a normal population. The test procedures include the likelihood ratio tests and the modified likelihood ratio tests. Admissibility of certain classification procedures are also proved.

1. Notation and preliminary results

The average of the columns of a matrix X will be denoted by $\bar{X}$. The parameter space will be denoted by $\Omega=\{\theta\}=H_0+H_1$ (or H_1+H_2). The entire random matrix will be denoted by V and its columns are defined to be independently distributed, each p-variate normal. *A priori* probability measure or positive constant multiples thereof will be denoted by Π. We only require $\Pi(\Omega)<\infty$. If $\Pi=\Pi_0+\Pi_1$ with Π_i a finite measure on H_i, we have the following lemma mentioned in Kiefer and Schwartz [1], where $f_v(v;\theta)$ denotes the density function of V:

Lemma 1.1. *Every Bayes critical region for* 0-1 *loss function is of the form*

$$\left\{v:\int f_V(v;\theta)\Pi_1(d\theta)\Big/\int f_V(v;\theta)\Pi_0(d\theta)>c\right\}\cup L_c\cup L \tag{1.1}$$

for some $c\,(0\leqq c\leqq\infty)$, *where* L_c *is a measurable subset of the set obtained from the set in brackets in* (1.1) *by replacing* $>$ *by* $=$, *and* L *is a measurable subset of the set* $M=\left\{v:\int f_V(v;\theta)\Pi(d\theta)=0\right\}$.

In our applications every L_c and L have probability zero for any θ in Ω, so that our Bayes procedures will be shown to be admissible by the following well-known lemma:

Lemma 1.2. *If a Bayes procedure is essentially unique for a problem with*