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On Stable Homotopy Types of Stunted Lens Spaces

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§1. Introduction

The purpose of this note is to prove some results on the stable homotopy types of the stunted lens spaces analogous to those in [8], [9] and [6].

The $(2n+1)$-dimensional standard lens space mod k is the orbit space

$$L^n(k)=S^{2n+1}/Z_k,\ Z_k=\{e^{2\pi li/k}\,|\,l=0,1,\cdots,k-1\},\ (n>0),$$

where the action is given by $z(z_0,\cdots,z_n)=(zz_0,\cdots,zz_n)$. Let $[z_0,\cdots,z_n]\in L^n(k)$ denote the class of $(z_0,\cdots,z_n)\in S^{2n+1}$. Imbed naturally $L^m(k)\subset L^n(k)$ by identifying $[z_0,\cdots,z_m]=[z_0,\cdots,z_m,0,\cdots,0]$ for $m\leqq n$, and consider the subspace

$$L_0^m(k)=\{[z_0,\cdots,z_m]\,|\,z_m \text{ is real}\geqq 0\}\subset L^m(k)\subset L^n(k).$$

Then $L^m(k)-L_0^m(k)$ and $L_0^m(k)-L^{m-1}(k)$ $(m\leqq n)$ are $(2m+1)$- and $2m$-cells which make $L^n(k)$ a finite CW-complex. The stunted spaces

$$L^n(k)/L^{m-1}(k),\ L^n(k)/L_0^m(k),\ L_0^n(k)/L^{m-1}(k) \text{ and } L_0^n(k)/L_0^m(k),$$

for $k=p^r$ where p is a prime and $n>m$, will be studied in this note.

We say that two spaces X and Y are stably homotopy equivalent (S-equivalent), if the suspensions S^aX and S^bY are homotopy equivalent for some a and b.

We obtain the following theorem which is [8, Th. A] when $r=1$.

THEOREM 1.1. *Let p be a prime and r a positive integer such that $p^r\neq 2$. If the stunted lens space $L^n(p^r)/L^{m-1}(p^r)$ is S-equivalent to $L^{n+t}(p^r)/L^{m-1+t}(p^r)$ for $n>m$, then*

$$t\equiv 0 \bmod p^{[(n-m-1)/(p-1)]}.$$

The same is true for $L^n(p^r)/L_0^m(p^r)$, $L_0^n(p^r)/L^m(p^r)$ and $L_0^n(p^r)/L_0^m(p^r)$.

For the case $p^r=2$, we have the following theorem which is proved