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K_{Λ}-Rings of Lens Spaces $L^n(4)$

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§1. Introduction

Let $L^n(k)=L^n(k;1,\cdots,1)$ be the $(2n+1)$-dimensional standard lens space mod k, where n and k are positive integers and $k\geqq 2$. Denote by Λ the field R of the real numbers or C of the complex numbers. The structure of K_{Λ}-rings of $L^n(k)$ is determined by J. F. Adams [1] when $k=2$ ($L^n(2)$ is the real projective space), and by T. Kambe [5] when k is an odd prime.

The purpose of this note is to determine the structure of $\tilde{K}_{\Lambda}(L^n(k))$ for the case $k=4$. We use K or KO instead of K_C or K_R.

Let η be the canonical complex line bundle over $L^n(k)$, and set

$$\sigma=\eta-1 \in \tilde{K}(L^n(k)).$$

Then, we have the following theorem[1]:

THEOREM A. (4.6)

$$\tilde{K}(L^n(4))\cong Z_{2^{n+1}}\oplus Z_{2^{[n/2]}}\oplus Z_{2^{[(n-1)/2]}},$$

and the direct summands are generated by the three elements

$$\sigma,\quad \sigma^2+2\sigma,\quad \sigma^3+2\sigma^2+2^{n/2+1}\sigma \qquad (\textit{if } n \textit{ is even}),$$

$$\sigma,\quad \sigma^2+2\sigma+2^{[n/2]+1}\sigma,\quad \sigma^3+2\sigma^2 \qquad (\textit{if } n \textit{ is odd}),$$

respectively. The multiplicative structure is given by

$$\sigma^4=-4\sigma^3-6\sigma^2-4\sigma,\quad \sigma^{n+1}=0.$$

Let ρ be the non-trivial (real) line bundle over $L^n(4)$ and set $\kappa=\rho-1 \in \widetilde{KO}(L^n(4))$. Let $r\sigma \in \widetilde{KO}(L^n(4))$ denote the real restriction of σ.

THEOREM B. (5.3, 5.6, 5.13, 5.18, 6.1, 6.7)

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1) According to N. Mahammed [8], it is announced that

$$K(L^n(k))\cong Z[\eta]/<(\eta-1)^{n+1}, \eta^k-1>$$

for any k.