HIROSHIMA MATH. J.
1 (1971), 155-159

The Unbounded Growth of Solutions of Parabolic Equations with Unbounded Coefficients

Lu-San CHEN*)

(Received May 18, 1971)

Dedicated to President Y.K. Tai on his 70th birthday

1. In 1966, Besala and Fife [1] studied the asymptotic behavior of solutions of the Cauchy problem for a parabolic differential operator

$$L=\sum_{i,j=1}^{n} a_{ij}\frac{\partial^2}{\partial x_i \partial x_j}+\sum_{i=1}^{n} b_i\frac{\partial}{\partial x_i}+c-\frac{\partial}{\partial t} \tag{1}$$

with non-negative Cauchy data not identically equal to zero. More recently Kuroda [4] also discussed an analogous problem under somewhat different conditions on the coefficients of such a parabolic differential operator and proved the following theorem:

Assume that the coefficients of (1) are defined for all $(x, t) \in R^n \times (0, \infty)$ and satisfy for some $\lambda \in (0, 1]$ the following hypotheses:

$$k_1(|x|^2+1)^{1-\lambda}|\xi|^2 \leqq \sum_{i,j=1}^{n} a_{ij}\xi_i\xi_j \leqq K_1(|x|^2+1)^{1-\lambda}|\xi|^2 \tag{2}$$

for any real vector $\xi=(\xi_1,\cdots,\xi_n) \in R^n$,

$$|b_i| \leqq K_2(|x|^2+1)^{1/2}, \quad 1\leqq i \leqq n, \tag{3}$$

$$-k_3(|x|^2+1)^{\lambda}+k_4 \leqq c \leqq K_3(|x|^2+1)^{\lambda}, \tag{4}$$

where $k_1(>0)$, K_1, $K_2(\geqq 0)$, $k_3(\geqq 0)$, $k_4(\geqq 0)$ and $K_3(>0)$ are constants.

Let the following inequality hold:

$$-2\left(\frac{r_0}{2\lambda\sqrt{k_1}}+\frac{nK_2}{4\lambda k_1}\right)\lambda K_1 n-4\left(\frac{r_0}{2\lambda\sqrt{k_1}}+\frac{nK_2}{4\lambda k_1}\right)^2\lambda^2 k_1+k_4>0 \tag{5}$$

where we have set $r_0=\left(k_3+\dfrac{n^2K_2^2}{4k_1}\right)^{1/2}$. If a non-negative function $u(x, t)$ continuous in $R^n \times [0, \infty)$ satisfies (i) $Lu \leqq 0$ in $R^n \times (0, \infty)$ in the usual sense, and (ii) $u(x, 0) \geqq 0$ and $u(x, 0) \not\equiv 0$ for $x \in R^n$ and $u(x, t) \geqq -\mu \exp(\nu(|x|^2+1)^{\lambda})$ for some positive constants μ and ν, then $u(x, t)$ grows to infinity exponentially as t tends to infinity and this exponential growth of $u(x, t)$ is uniform in any compact subset of R^n.

*) This research was supported by the National Science Council.