HIROSHIMA MATH. J.
1 (1971), 145-153

On the Behavior of Solutions of the Cauchy Problem for Parabolic Equations with Unbounded Coefficients*)

Lu-San CHEN, Cheh-Chih YEH and Hung-Yih CHEN

(Received May 18, 1971)

Dedicated to President Y.K. Tai on his 70th birthday

1. Let $x=(x_1,\cdots, x_n)$ be a point of the n-dimensional Euclidean space R^n and let t be a non-negative number. The distance of the point $x\in R^n$ from the origin of R^n is denoted by $|x|=(\sum_{i=1}^{n} x_i^2)^{1/2}$. The $(n+1)$-dimensional Euclidean half space $R^n\times(0, \infty)$ is the domain of interest.

Consider a parabolic differential equation

$$(1)\qquad L_0u=\sum_{n=1}^{n}\frac{\partial^2 u}{\partial x_i^2}+(-k^2|x|^2+l)u-\frac{\partial u}{\partial t}=0,\ (k>0)$$

in $R^n\times(0, \infty)$. Krzyżański [4] proved the existence of the fundamental solution of this equation. By using this fundamental solution, we can see that the solution $u(x, t)$ of the above equation with Cauchy data $u(x, 0)=$ Mexp $(a|x|^2)$ $(2a<k)$ is given by

$$u(x, t)=M\left(\frac{k}{k\cosh 2kt-2a\sinh 2kt}\right)^{n/2}$$

$$\exp\left[\frac{k(2a\cosh 2kt-k\sinh 2kt)}{2(k\cosh 2kt-2a\sinh 2kt)}|x|^2+lt\right].$$

So, if $l-kn<0$, then $u(x, t)$ converges to zero uniformly on every compact set in R^n as $t\rightarrow\infty$, (cf. [7]). This fact leads us to the question whether the similar situation to the above holds or not for solutions of general parabolic equations of unbounded coefficients with suitable Cauchy data.

2. The following results, Theorem A and Theorem B, of Kusano [8] give us an answer to the question.

Let

$$(2)\qquad Lu=\sum_{i,j=1}^{n}a_{ij}\frac{\partial^2 u}{\partial x_i\partial x_j}+\sum_{i=1}^{n}b_i\frac{\partial u}{\partial x_i}+cu-\frac{\partial u}{\partial t}=0$$

be a parabolic differential equation in $R^n\times(0, \infty)$, where the coefficients $a_{ij}(=a_{ji})$, b_i and c are functions defined in $R^n\times[0, \infty)$ and such that

*) This research was supported by the National Science Council.