Corrections to "The Reduced Symmetric Product of a Complex Projective Space and the Embedding Problem"

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There is a mistake in §5 of my previous note [6], and the results (2) and (3) of theorem 5.5 on pages 28 and 39 are incorrect. This theorem should be replaced by

THEOREM 5.5. Let $n \ge 4$.

(1) There exists a unique isotopy class of embeddings of CP^n in R^{4n} .

(2) There exist countable isotopy classes of embeddings of CP^n in \mathbb{R}^{4n-1} .

(3) There exists a unique isotopy class of embeddings of CP^n in R^{4n-2} for $n \neq 2^r$.

This note contains some corrections of $[6, \S5]$ and the proof of (2) and (3) of the above.

1. Some Corrections. In this note, denote simply by SZ the quotient manifold $SZ_{n+1,2}$ of [6. (1.3)] and let λ be the real line bundle associated with the double covering $Z_{n+1,2} \longrightarrow SZ$. (In $[6, \S5]$, we consider also λ as the real line bundle associated with the double covering $CP^n \times CP^n - \Delta \longrightarrow (CP^n)^*$.) Let \mathscr{B} be the S^{m-1} -bundle associated with $m\lambda$ and let $\mathscr{B}(\pi_i(S^{m-1}))$ be the bundle of coefficients with fiber $\pi_i(S^{m-1})$ associated with \mathscr{B} . Then the obstructions for the existence of a non-zero cross section of $m\lambda$ are the elements of $H^{i+1}(SZ; \mathscr{B}(\pi_i(S^{m-1})))$ and the obstructions for two given non-zero cross sections being homotopic are the elements of $H^i(SZ; \mathscr{B}(\pi_i(S^{m-1})))$. If m is even, then the bundle of coefficients $\mathscr{B}(\pi_i(S^{m-1}))$ is trivial since $m\lambda$ is orientable, and so the above cohomology groups with local coefficients coincide with the ordinary cohomology groups.

Therefore the cohomology groups $H^*(SZ; \pi_i(S^{m-1}))$ for odd m in [6, §5, pp. 38-39] should be replaced by $H^*(SZ; \mathscr{B}(\pi_i(S^{m-1})))$.

2. PROOF OF THEOREM 5.5. (2). By $[4, \S37.5]$ and [6, Prop. 5.2(2)], it is sufficient to show that $H^{4n-2}(SZ; \mathscr{B}(\pi_{4n-2}(S^{4n-2})))=Z$. Since $(4n-1)\lambda$ is unorientable, the bundle of coefficients $\mathscr{B}(\pi_{4n-2}(S^{4n-2}))$ with fiber $\pi_{4n-2}(S^{4n-2})=Z$ is not trivial by $[4, \S38. 12]$. Let \mathscr{B}' be the tangent sphere bundle of SZ. Because SZ is a (4n-2)-dimensional unorientable manifold by [6, Th. 4.15], the bundle of coefficients $\mathscr{B}'(\pi_{4n-3}(S^{4n-3}))$ with fiber $\pi_{4n-3}(S^{4n-3})=Z$ is not