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On Width Ideals of a Module

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The notion of width of a module was introduced by M.-P. Brameret and some properties of it were shown in the paper [1]. Moreover M. Wichman obtained some results on this subject in the case of modules over a commutative ring in [4]. On the other hand H. Fitting studied the determinantal ideals of a finitely generated module over a commutative ring for the first time in [2] and several authors used this notion for the study of modules. In particular it was shown by T. Matsuoka in [3] that some properties of the torsion submodule of a module have a close connection with Fitting's determinantal ideals.

The aim of this note is to show relations between these two notions. For this purpose we give the notion of weak width of a module over a commutative ring which is more fitting for us than that of width of a module, and elementary properties of it are shown. Next we define the width ideals of a module and show that these ideals are natural modifications of Fitting's determinantal ideals for a not necessarily finitely generated module. Moreover it is shown that the weak width of a module over an integral domain has a close connection with width ideals or Fitting's determinantal ideals of the module. Lastly we shall give a generalization of the results on the torsion submodule of a module in [3].

Throughout this paper all rings will be commutative with unit and all modules will be unitary.

§ 1. Weak width of a module

Let R be a commutative ring with unit and U the set of regular elements of R[1]. Let M be an R-module. Then we understand by the *weak width* $W'(R, M)$ *of* M *over* R the smallest integer n such that for any set $\{x_1, \cdots, x_{n+1}\}$ of $n+1$ elements of M, we have a soluution $ax_i=\sum_{j\neq i} a_j x_j$ for some i, a in U and a_j in R. In other words $W'(R, M)$ is the width $W(R_U, M_U)$ of M_U over R_U in the sence of [1]. If $W'(R, M)=n$, there exists a set $\{x_1, \cdots, x_n\}$ of n elements of M such that ax_i is not contained in $\sum_{j\neq i} Rx_j$ for any i and any a in U. We call a system with the above property *a set of*

1) An element of a ring R is called regular, if it is not a zero-divisor of R. If an ideal of R contains a regular element of R, it is called a regular ideal.